ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII „ALEXANDRU IOAN CUZA" DIN IAŞI
Tomul LVI Ştiinţe Economice 2009

THE USE IN BANKS OF VALUE AT RISK METHOD IN MARKET RISK MANAGEMENT

Ioan TRENCA*

Abstract

In sophisticated market environments, banks with sufficient liquidity can normally hedge against market volatility. The resulting net effective open position determines the amount of the portfolio that remains exposed to market risk, which Value at Risk can measure. In contrast with traditional risk measures, VaR provides an aggregate view of a portfolio's risk that accounts for advantage, correlations, and current positions. As a result, it is truly a forward-looking risk measure that applies not only to derivatives but also to all financial instruments. Furthermore, the methodology can also be broadened from market risk to other types of financial risk, using Delta-Normal Method, Historical Simulation, or Monte Carlo Simulation.

Key words: Value at Risk method, market risk management, market volatility, financial risk, portfolio's risk
JEL classification: G21, G32

Value at Risk is the methodology used to estimate the market risk to which a bank is exposed, and also for determining, the banks' minimum capital required to cover this risk. It measures the maximum loss likely to be lost in a portfolio in a given period, and for a given confidence interval.

VaR methodology was born in 1994, when the President of „J.P. Morgan Investment Bank", Dennis Weatherstone, asked employees to submit a report every day about the bank's degree of risk together with a corresponding risk measure. Thus in October 1994, the well-known department of Risk Metrics was established within the bank, specialized in the risk study and analysis. The risk measure used has become known under the name of VaR. It is currently used worldwide by many banks, investment funds, brokerage firms, and non-financial companies. Value at Risk is the final step in the evolution of risk management instruments, combining the relationship between price and performance with the probability of unanticipated market movements. It takes into account the correlations between financial assets of the portfolio and the advantage effect. This has a dual role, both for measuring market risk on an integrated basis, and determining the minimum capital required to cover the banks' market risk. In a model based on VaR, data on bank positions, prices, volatility, and risk factors are introduced. Risks covered by the model must include all items of inter-

* **Ioan TRENCA** (ioan.trenca@econ.ubbcluj.ro**),** PhD, Professor, "Babes-Bolyai" University of Cluj-Napoca., Faculty of Economics and Business Management.

est, shares, commodities, options and foreign exchange positions, balance sheet and off-balance sheets.

Provisions of the Basel II Agreement

In recognition of the increasingly large banks exposure to market risk, and in order to take advantage of the discipline imposed by capital requirements, the Basel Committee on Banking Supervision Issues promoted the 1988 Capital Agreement, adding in January 1996 capital expenditure specific to market risk. It includes a set of quantitative and qualitative standards for the process of risk management that are applied to banks using internal models. Thus, models of internal risk evaluation, used by banks, enter into a common conceptual framework, assessing aggregate exposure to market risk of the entire portfolio.

The 1996 Amendment to the Basel II has brought an added capital margin for the market risk, including for the bank's trading portfolio (trading book - composed of financial instruments owned for a short term to be sold, which are market marked) and for other portfolios consisting of financial instruments, mainly credits (banking book). To estimate the market risk, banks may use the standard method, together with the internal models for determining the VaR. Internal models are more advantageous for large banks, as they take into account the correlations between assets, and require a much lower capital cost. The assessment of market risk through the VaR method, respecting the Basel II provisions, must meet certain conditions:

- the daily evaluation of the market risk related to the interest rates and capital instruments of the transactions portfolio;
- the daily assessment of the currency risk rate of the Bank's portfolio;
- use a trust level of 99% for the VaR calculation;
- use an instant price shock equivalent to a price associated movement of 10 days;
- use a historical observation period of at least one year;
- updating of data sets at least once every 3 months and their reassessment whenever market prices change;
- recognition of empirical correlations between major risk categories such as interest rates, exchange rates, prices of capital instruments and goods, including the volatility of options in each category of risk factors
- the possibility of carrying out operations of stress-testing and back-testing;
- establishing and monitoring VaR limits;
- establishing a separate capital margin to cover the risk of specific interest rates and capital instruments.

Standard capital market risk of the Basel Committee requires that VaR be calculated daily and capital requirements related to market risk are met daily. Capital requirements are expressed as the maximum value of the previous day's VaR and the average of daily VaR indicators for the last 60 working days. This is then multiplied by an additional multiplier k (whose minimum value is 3) designated by the national supervisory authorities according to the quality of risk management related to the banks owned portfolio.

Thus, the margin of market risk related to moment *t* is:

$$MRP_t = \max(k \frac{1}{60} \sum_{i-1}^{60} VaR_{t-1}, VaR_{t-1}) + MRS_t ,\qquad \text{where:}$$

MRS- represents the specific margin risk to each title in the portfolio, varying according to the sensitivity of each title to the market changes. *K* was created in order to provide additional protection to banks that are not very strong and banks operating in an unstable market.

Building the Value at Risk Model

Value at risk measures the probable maximum loss registered on a certain position or a positions' portfolio in a given period and for a given confidence interval. The biggest advantage of using this methodology is that a single number summarizes the bank's exposure to several risk variables.

To determine VaR the following parameters should be set:

Time horizon for the risk (t): it depends on the risk factors and the maturity of the portfolio positions. For more accurate risk measurement of the established benchmark, it is recommended the calculation of VaR on a short time horizon; for the bank capital adequacy in relation to market risk exposure, it is recommended to use a longer interval.

Confidence probability (α) or *percentage of risk tolerance (1 - α)* should reflect the bank's aversion towards the capital cost that will exceed VaR. Greater aversion to risk, and the cost of higher capital adequacy will lead to the establishment of a high level of confidence. It is recommended that it maintain within the margin of 95% -99%, if not, VaR accuracy will suffer.

Corresponding VaR confidence level α is given by the smallest number l such that the probability of loss L to exceed l is not greater than 1 - α, as follows:

$$VaR_{\alpha} = \inf\{l \in \Re : P(L > l) \le 1-\alpha\} = \inf\{l \in \Re : F_L(l) \ge \alpha\}$$

VaR is determined as the difference between the expected value of the portfolio in the chosen time horizon ¨ t ¨ with a certain probability ¨ p ¨ and the lowest value of the portfolio (given by the level), at the same time horizon and with the same probability, as follows:

$$VaR = V_m - V_q = V_0(1+R_m) - V_0(1+R_q) = V_0(R_m - R_q)$$, where:

V_0 - current market value of portfolio;

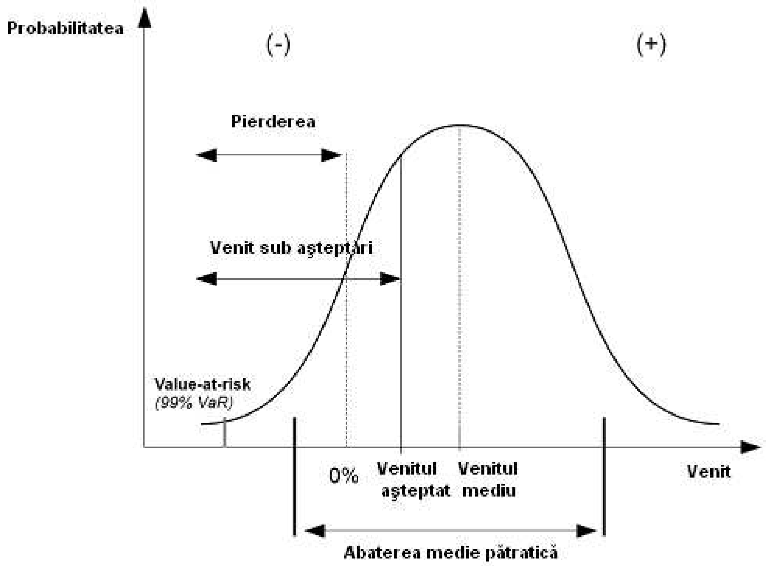
V_m - expected value of the portfolio on the time horizon t

V_q - the lowest value that a portfolio can record (level) on the chosen time horizon, corresponding with the confidence interval;

R_m - the portfolio's average yield on the time horizon t;

R_q - yield corresponding level

Value at risk graph can be represented as follows:



The lowest value of the portfolio return at the chosen time horizon ¨ t ¨ with a certain probability ¨ α ¨ is determined from the distribution of return:

$$1-\alpha = P(x < R_q) = \int_{-\infty}^{R_q} f(x)dx \cdot$$

If the cumulative distribution function is unknown, and especially if it is the cumulative normal distribution, then the corresponding yield level can be determined by the relationship:

$$R_q = \alpha \times \sigma + R_m$$

Starting from the previous relationship, VaR formula is obtained assuming normal distribution:

$VaR = V_m - V_q = V_0(R_m - R_q) = V_0\left[R_m - (\alpha \times \sigma + R_m)\right] = -V_0 \times \alpha \times \sigma$, where,

V_0 - Current market value of portfolio

α - level of trust;

σ- portfolio volatility

It is noted that an increase in portfolio volatility will lead to the flattening of the yield distribution curve, which will increase VaR. If the period of detention is short, changing the average yield will not have a significant impact on the calculation of VaR, but if the period of detention is high, the changes of average profitability will be significant. VaR will also increase along with the confidence levels.

VaR is a method often used, but imperfect, being quite difficult to estimate. Sometimes this can lead to false conclusions, because it underestimates the frequency of small return. In addition, the indicator is very difficult to determine for a bank with a much-diversified portfolio of titles. Therefore, banks should always test the accuracy of VaR methodology through stress-testing and back-testing tests, especially if VaR is used by banks for capital adequacy to market risks.

Alternatives to VaR

Due to limitations of VaR, various alternatives have developed. One of them is *Conditional VaR*, which can cause an expected portfolio loss when VaR is exceeded, as follows:

$$P(x<q)=\int_{-\infty}^{q} xf(x)/\int_{-\infty}^{q} f(x)$$

Conditional VaR is a superior VaR measure for quantifying market risk that can be used successfully to optimize market portfolios of banks, regardless of whether it follows a normal distribution law, or not. The main disadvantage of the method is that it does not allow the effective implementation of the back testing methods.

Another alternative to VaR is the *Marginal VaR*, which may be a factor in the decision to incorporate or not a new title in the portfolio, assessing the marginal contribution of the new title relating to the whole portfolio VaR. This value is determined by calculating VaR sensitivity value invested in the portfolio’s component *i*.

Incremental VaR represents the VaR effect on a new transaction. If a portfolio component is sufficiently small in relation to the portfolio value, it might be considered that marginal VaR remains constant as the value of x_i tends to 0. It is determined as the difference between VaR related to the initial portfolio position (VaR_p) and the VaR related to the new portfolio positions (VaR_{p+a}):

$$VaR_{incremental}=VaR_{p+a}-VaR_p$$

If the VaR decreases, then the new transaction will reduce the risk of the portfolio, or, on the contrary, will enhance it. The *a* coefficient represents a change in one or more components of the portfolio, in which it must be taken into account their aggregate effect Difference with marginal VaR is that it can quantify a larger modification in the portfolio composition. Although it is difficult to implement because it involves a total revaluation of the portfolio, the method successfully applies where a new transaction involves portfolio exposure to new risks. If *a* is the new value that will be invested in asset *i*, and W is the present value of the portfolio, then the risk of the new portfolio will be given by the following relationship:

$$\sigma_{pf1}^2=W^2\sigma_{pf0}^2+a^2\sigma_i^2+2Wa\sigma_{pf0,i}$$

To determine the size of new transactions leading to minimize the portfolio risk, the derived of order I of the latter expression in report with *a* will be determined:

$$\frac{\partial\sigma_{pf1}^2}{\partial a}=2a\sigma_i^2+2W\sigma_{pf0,i}$$

Equaling to zero this expression, we will obtain: $a=-W\frac{\sigma_{pf0,i}}{\sigma_i^2}=-W\beta_i\frac{\sigma_{pf0}^2}{\sigma_i^2}$

Value at Risk Models

VaR based models combine the potential modification of each position, resulting from specific risk factors variations, with the probability of such variations. Value changes are aggregated to segments level from the transaction registry, and to the level of trading markets.

VaR value can be calculated using the following methodologies:

- the Analytical method (co-variation- variation method or delta-normal);
- historical simulation method;
- Monte Carlo simulation method.

Analytical method involves assuming a normal distribution of portfolio securities values yields. Profitability is considered gradually independent, not being influenced by the previous day's profitability. To calculate the potential modification of current portfolio value, it is calculated the average and standard deviation of portfolio titles return in order to achieve a combination of individual positions sensitivity to risk factors from the co-variation matrix, representing the risk factors volatility and the correlations between assets. It is one of the most easily implemented methodologies; it also presents some disadvantages, because the normal assumption on which it is based is rather rarely met in practice. Most distributions actually have oblong tails (fat tails); characterized by a large number of unforeseen events, in which case VaR can not estimate well the large losses. On the other hand, volatility and correlation coefficients are variable in time, having a significant impact especially if portfolios contain options

Historical Simulation Method calculates the hypothetical value of a change in the current portfolio depending on historical variations of the risk factors. The great advantage of the method is that it makes no assumption regarding the distribution of profitability, using the empirical distribution obtained from analysis of past data, while being a relatively simple calculation. Because is not dependent on assumptions regarding the parameters of the markets evolution, this methodology can be adapted to leptokurtic, asymmetric and other abnormal distributions. The disadvantage of the method lies in the fact that it predicts the future development based on past data, which could lead to inaccurate forecasts if the trend of the past no longer complies, or if the portfolio changes.

In the case of *Simulation Monte Carlo Method,* the distribution of portfolio return is obtained by generating different scenarios for the considered risk factors, and calculating the portfolio value in these circumstances. The method is flexible and can be applied to all types of portfolios, but requires a larger power of calculation and the careful choice of evaluation models for portfolio's financial assets.

If a large enough number of possible profit or loss values recorded by the portfolio is simulated, then it can build a probability density, generating the VaR based on the lowest percentile of the distribution. The first step is to choose a stochastic model for the behavior of prices, one of the most frequently used being the *geometric Brownian motion*. This implies that prices of financial assets are not correlated over time, the variant decreasing as the time increases. The change of the value of portfolio assets may be described as follows:

$dS_t = \mu_t S_t dt + \sigma_t S_t dw$, where

St is the value of financial assets; μt is the expected yield per unit of time; σt is the financial asset volatility;

DW is a Wiener process, which can be written as $dw = \varphi(dt)^{\frac{1}{2}}$, where φ is a random variable with a standard normal distribution.

Instant return of financial assets varies depending on the trend μtdt and the random variable μt, in practice the discrete model being used. Thus, if Δt is the frequency with which the asset return is measured, and ΔS is change in price in the time interval Δt, then we have:

$$\frac{\Delta S}{S_t} = \mu_t \Delta t + \sigma_t \varphi \sqrt{\Delta t}$$

Thus, the financial assets yield is considered to have a normal distribution with average $\mu_t \Delta t$ and standard deviation. Value at risk of the portfolio titles will then determine the distribution of action price at the time T (S (T)).

Testing model accuracy

The Basel Committee recommends that banks use regularly rigorous stress-testing programs to identify events that could have a negative impact on the bank's capital position. Stress tests should have both a qualitative and a quantitative nature. Quantitative criteria should identify plausible stress scenarios that might arise in the market. Quality criteria aim assessing the Bank's ability to absorb big losses, and the measures the bank can take to reduce risk. Stress-testing methodology involves several steps:

- review information on the actual highest losses recorded during a given period, compared with the estimated losses from the bank's internal risk assessment;
- simulation of extreme stress scenarios, by incorporating both the large price variations, and large reductions in the level of liquidity that are associated with these events;
- evaluate the degree of banks exposure sensitivity to market risk from changing assumptions on volatility and correlations;
- resumption of bank's specific stress scenarios, which surprise the characteristics of the trading portfolio of a bank in the most adverse conditions.

In addition, banks are recommended also to use back testing, which is based on testing a sample of data from 250 days ago until the day on which VaR is calculated. Through it, they are trying to determine how often and by what amount the VaR limit has been exceeded. Banks can use two approaches:

the binary loss function approach-which helps determine the factor k for capital adequacy. The test can be described as follows:

$$T_i = \begin{cases} 1, pierderea_i < VaR_i \\ 0, pierderea_i \geq VaR_i \end{cases}$$

The test result is:

- the maximum number of exceedances of the VaR on a horizon of 250 days supported by the Basel Committee is 4, if not, the model used in the calculation of VaR is not suitable.
- *the quadratic loss function approach* –is used to compare different VaR models, and consists in the following test, where P is the portfolio loss :

$$T_i = \begin{cases} 1 + (\Delta P_t - VaR_t)^2, \Delta P_t < VaR_t \\ 0, \Delta P_t \geq VaR_t \end{cases}$$

The test result is $T = \sum_i T_i$.

Applying the methodology of VaR on the BCR portfolio rate using the analytical method

In the analysis of exposure to currency risk, BCR uses VaR methodology, being identified long and short positions held by each bank on its portfolio of foreign currency. On 28.12.2007, the Bank had the following currency structure of the portfolio, consisting of ten currencies:

Table no. 1. Currencies Portfolio held by the bank

Currency	Current position	Exchange rate (28.12.2007)	Long Position	Short Position
AUD	11361,123	2,15370		24468,450
CAD	-71545,175	2,50800	179435,300	
CHF	18754,944	2,17440		40780,750
DKK	-16848,069	0,48410	8156,150	
EUR	15543338,873	3,61020		56114562,000
GBP	-39871,189	4,90950	195747,600	
HUF	57236,140	1,42500		81561,500
JPY	-112415,924	2,17660	244684,500	
NOK	72082,634	0,45260		32624,600
USD	3132861,505	2,45640		7695561,000

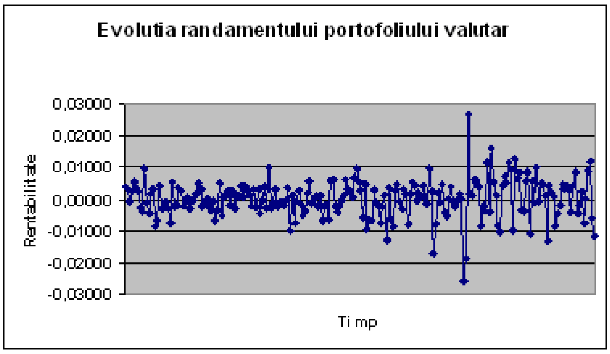
In the next step, it has been taken into account the exchange rates of ten of the portfolio currencies over a period of 225 days (from 03.01.2007 to 28.12.2007), calculating continuously their daily return, through the logarithmic method, and the volatility, through average square deviation. The data obtained are summarized in the following table:

Table no. 2. Daily Return and Volatility

Currency	**AUD**	**CAD**	**CHF**	**DKK**	**EUR**
Average	0,00025	0,00061	-0,00003	0,00008	0,00007
Deviation	0,00588	0,00629	0,00520	0,00376	0,00376

Currency	**GBP**	**HUF**	**JPY**	**NOK**	**USD**
Average	-0,00007	0,00003	-0,00007	0,00031	-0,00040
Deviation	0,00430	0,00408	0,00864	0,00441	0,00571

Daily evolution of the portfolio return rate is graphically represented as follows:



The next step is to determine correlation coefficients between portfolio currencies and the construction of the correlation coefficients matrix corresponding to all currencies of the portfolio. Calculation of correlation coefficients is represented in the following formula:

$$\rho_i = \frac{\sigma_{ij}}{\sigma_i \times \sigma_j};$$

$$\sigma_{i,j} = \left[\sum_{t=1}^{T}(R_{i,t} - \overline{R_i}) \times (R_{j,t} - \overline{R_j})\right] / T - 1; \; i = \overline{1,n}, \; j = \overline{1,n}$$

Table no. 3. The matrix of correlation coefficients

	AUD	CAD	CHF	DKK	EUR	GBP	HUF	JPY	NOK	USD
AUD	1,00000	0,45264	0,02293	0,25063	0,25310	0,33110	0,34712	-0,1081	0,31308	0,18832
CAD	0,45264	1,00000	0,38949	0,56903	0,56514	0,50512	0,33049	0,34910	0,45393	0,62768
CHF	0,02293	0,38949	1,00000	0,88376	0,88542	0,56669	0,03184	0,87460	0,52683	0,70920
DKK	0,25063	0,56903	0,88376	1,00000	0,99889	0,72323	0,25584	0,76516	0,69285	0,81467
EUR	0,25310	0,56514	0,88542	0,99889	1,00000	0,72144	0,25829	0,76558	0,69469	0,81283
GBP	0,33110	0,50512	0,56669	0,72323	0,72144	1,00000	0,26505	0,52995	0,53899	0,71382
HUF	0,34712	0,33049	0,03184	0,25584	0,25829	0,26505	1,00000	-0,0940	0,32901	0,11362
JPY	-0,1081	0,34910	0,87460	0,76516	0,76558	0,52995	-0,0940	1,00000	0,40113	0,77237
NOK	0,31308	0,45393	0,52683	0,69285	0,69469	0,53899	0,32901	0,40113	1,00000	0,50857
USD	0,18832	0,62768	0,70920	0,81467	0,81283	0,71382	0,11362	0,77237	0,50857	1,00000

In determining the probability with which to calculate the maximum loss related to the bank's portfolio, it is used a confidence coefficient α = 2.33 corresponding to a probability of 99%, which is also the recommendation and BNR To calculate the daily VaR, corresponding to each currency in the portfolio, the following relationship will be used:

$$VaR_i = -V_{i,0} \times \alpha \times \sigma_i \text{, where:}$$

$V_{i,0}$ - represents the current market value of the bank's exposure in each currency portfolio on 29.12.2007;

σ_i - volatility of each currency in the portfolio

The previous formula will determine the maximum possible losses related to foreign currencies in each portfolio.

Daily maximum possible losses:

Currency	Volatility	Net Position	Daily VaR (99%)
AUD	0,0059	-24468,4500	335,1354
CAD	0,0063	179435,3000	-2627,6752
CHF	0,0052	-40780,7500	493,9318
DKK	0,0038	8156,1500	-71,3786
EUR	0,0038	-56114562,0000	492018,0301
GBP	0,0043	195747,6000	-1963,1482
HUF	0,0041	-81561,5000	776,0326
JPY	0,0086	244684,5000	-4923,4445
NOK	0,0044	-32624,6000	335,0248
USD	0,0057	-7695561,0000	102352,0702

It is noted from the table that the largest possible daily loss (492018.0301 billion) may be caused by holding a short position in EUR, which was followed by holding a short position on USD; the maximum possible loss that could record in this case is 102352.0702. The maximum possible loss if the currency would be uncorrelated would be the amount of losses for each individual currency, recording a value of 586,724.58 lei. Because currencies are correlated, it is necessary to calculate the daily VaR indicator for the currency portfolio of the bank, taking into account the correlation between the currencies presented in the matrix of correlation coefficients, as follows:

$$VaR_{pf}^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} VaR_i \times VaR_j \times \rho_{ij}$$

$$VaR_{pf}^2 = (VaR_1 \quad VaR_2 \quad \dots \quad VaR_n) \times \begin{pmatrix} \rho_{11} & \rho_{12} & \dots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \dots & \rho_{2n} \\ \dots & \dots & \dots & \dots \\ \rho_{n1} & \rho_{n2} & \dots & \rho_{nn} \end{pmatrix} \times \begin{pmatrix} VaR_1 \\ VaR_2 \\ \dots \\ VaR_n \end{pmatrix}$$

It is obtained a value of VaR indicator corresponding to the currency portfolio, for a day, of 112315.7089 lei. To determine the maximum possible loss that can be recorded on a horizon of 10 days (h), the following formula will be applied:

$$VaR_{pf,h} = VaR_{pf} * \sqrt{h}$$

achieving a value of 355173.4571 lei. It is noted that calculated VaR, taking account of correlations between currencies, is lower than the VaR calculated by individual aggregating VaR indicators for each currency separately.

The data obtained can be summarized as follows:

VaR for 1 day	112.315,7089 lei
Probability	99%
Level of confidence	2,33
Time horizon	10 days
VaR for 10 days	355.173,4571 lei.

Conclusions

Value at risk is the most used method of quantifying the market risk, being also a measure for determining minimum capital limit required for banks to cover the exposed market risk. This limit is prescribed by the Basel II through a set of quantitative and qualitative requirements. Estimating the maximum loss of a financial instruments portfolio, this method involves the arbitrary choice of two parameters: time horizon and rate of risk tolerance.

Because it measures with a certain error, the risk exposure, as of the confidence percentage and used simplifications, various alternatives to VaR have developed: Conditional VaR, Marginal VaR and VaR Incremental. In practice, there are used several methods for determining the indicator, the best-known are: parametric method, the historical simulation and Monte Carlo simulation method. In choosing one of the methods, it must be taken into account the accuracy and speed of each model. Parametric method is simple, but is based on the assumption of normality. Historical method is easily implemented, but does not accurately capture the risk of future events. The most powerful of them is the Monte Carlo simulation, which requires a power calculation measure.

References

Greuning, H., Bratanovic S., *Analyzing Banking Risk, A Framework for Assesing Corporate Governance and Financial Risk*, The World Bank, Editura Irecson, Bucureşti, 2004

Governance and Financial Risk, The World Bank, Editura Irecson, Bucureşti

Hull J., *Options, Futures and Other Derivatives*, Editura Prentice Hall, USA, 1999

Jackel P., *Monte Carlo methods in finance*, Editura John Wiley&Sons, USA, 2002

Jorion P. Value at Risk: the new benchmark for managing financial risk, Editura McGraw-Hill, USA, 2001

Jorion P., *Financial Risk Manager Handbook*, Editura Wiley&Sons, England, 2003

Angelidis T., Mau R., Shenoy C., A robust VaR model under different time periods and weighting schemes, Review of Quantitative Finance and Accounting, Boston, vol. 128, pg. 197

Basel Committee on Banking Supervision, *The New Basel Accord*, 2003, http://www.bis.org/bcbs/qis/index.htm

Chipalkatti N., Datar V., The relevance of value-at-risk disclosures: evidence from the LTCM crisis, Journal of Financial Regulation and Compliance, London, vol. 14, pg. 174

Shijin S., Kumar A., Bhattacharyya S., *The relationship between size, value, and market risk: some evidence, Investment Management & Financial Innovations*, vol. 4, pg. 125

Yakov Ben-HaiM, *Value-at-risk with info-gap uncertainty*, The Journal of risk finance, 2005;6;5, pg. 388