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A SMALL OPEN INTERREGIONAL MONETARY SPATIAL ECONOMIC GROWTH WITH THE MIU APPROACH

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Abstract

We build an interregional monetary economic growth model with housing and environment for an open economy in a perfectly competitive economy. The monetary aspects of the model are based on the money-in-utility-function (MIU) approach. The real aspects of the model are influenced by the three most well-known models in the growth theory and urban economics in a multi-regional context. The industrial production follows the Solow model. Regional population distributes over the urban area according to the Alonso model and regional housing production follows the Muth model. We accept a new approach to the microeconomic foundation for determining money holding, savings, spatial location and consumption. Following the traditional literature of small open economies, we assume that the rate of interest is fixed in international market. We explicitly solve the equilibrium of the multi-regional economy. We also simulate the model and examine effects of changes in the rate of interest, the inflation policy, the preference, and amenity parameters on the long-term equilibrium structure of the multiregional spatial economy.

Key words: model, economic growth, open economy
JEL classification: E42, P44

1. Introduction

The purpose of this study is to introduce money into an interregional growth model. There is only a few studies with microeconomic foundation which explicitly deal with issues related to interactions among regional economic development, land and housing rents and spatial environment. As systematically examined by Zhang (2009), forms of money and its functions exhibit a great variety, depending on, for instance, technology, economic developmental stages, institutional structures, and man's attitude towards the future. Money does not only provide services of the present but also plays the role of a connector of the present and the future. Many of the most intriguing and important questions in dynamic theory involve money. Nevertheless, many of economic dynamic models omit monetary issues, by explicitly or implicitly assuming that transactions on the economy's real side can be carried out frictionlessly without money. Modern analysis of the long-term interaction of inflation and capital formation begins with Tobin's seminal contribution (Tobin, 1965)[1]. In Tobin's approach, a monetary economy has a real sector exactly like that in the Solow growth model, so that the monetary nature of the model depends on how money is introduced into the

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model. As a depositor of purchasing power money can be held by private agents as an alternative form of wealth to physical capital stock. Tobin showed that an increase in the level of the inflation rate will increase the capital stock of an economy. Sidrauski (1967) constructed an economic model in which no real variable will be affected by the economy's inflation rate. We will address the issues by Tobin and Sidrauski in the alternative framework. Our approach is strongly influenced by the money-in-utility (MIU) function approach which was initially proposed by Patinkin (1965) and Sidrauski (1967). In this approach, money is held because it yields some services and the way to model it is to enter real balances directly into the utility function. This approach has been widely applied in monetary growth theory (for instance, Brock, 1974; Turnovsky, 2000).

Different from the traditional monetary growth models, this study is explicitly concerned with interregional differences in the role of money on economic growth. There should be close interactions among money, housing market and economic development. Lucas (1988) points out the necessity of analyzing urban configuration and economic growth as a connected whole. However, partial equilibrium models fail to explain interactions among various economic activities over time and space. We try to integrate some well-known models in a compact analytical framework. Our approach to economic growth is strongly influenced by the neoclassical growth theory.[1] There is a large literature on durable housing in spatial context.[2] It is important to develop analytical frameworks for examining spatial economic growth with money, housing, transportation systems, and environment. Nevertheless, only a few economic models study issues related to growth and residential distribution, not to mention to include the role of money in economic dynamics, within a comprehensive framework. In particular, contemporary formal economic growth theory with capital accumulation has almost neglected housing markets. Over the past three decades, the study of the economic growth with housing and economic geography has caused a lot of attention in urban economics and regional science.[3] Yet, it is argued that urban economics still needs an analytical framework for spatial evolution and growth with capital accumulation. Zhang (2008b) has proposed a model which synthesizes the neoclassical growth theory, the Alonso residential location model and Muth housing model. This study is an extension of Zhang's model in that money is explicitly introduced to the spatial economy. This study describes dynamics of a small country economy with economic geography. There are many economic models concerned with growth and capital accumulation of small open economies.[4] Nevertheless, this analytical framework has not been extended to spatial economics with land use and housing market, except Zhang's recent interregional monetary growth model (Zhang, 2008a). This study is different from Zhang's interregional monetary model in that this study explicitly introduces spatial economic structure into the interregional economy. We see that this study is a synthesis of Zhang's previous two models. This paper differs from the previous studies mainly in that this model is concerned with an interregional

[1] Most of the models in the neoclassical growth theory are extensions and generalizations of the pioneering works of Solow (1956). The Solow model is sometimes referred as to the Solow-Swan model because Swan (1956) proposed a model similar to the Solow model. The literature of the neoclassical growth theory is referred to Burmeister and Dobell (1970).

[2] See, for instance, Muth (1973), Anas (1978, 1982), Hockman and Pines (1980), Brueckner (1981), Arnott (1987), Brueckner and Pereira (1994), Arnott et al (1999), Braid (2001), and Brito and Pereira (2002).

[3] We refer on the literature surveys to Henderson and Thisse (2004), Capello and Nijkamp (2004), and Andersson and Johansson (2007).

[4] Refer to, for instance, Obstfeld and Rogoff (1999), Lane (2001), Kollmann (2001, 2002), Benigno and Benigno (2003), and Galí and Monacelli (2005), for the literature on economics of open economies.

monetary economy with growth urban structure, while the interregional model by Zhang (2008a) is concerned with an economy without urban structure and housing market and the urban model by Zhang (2008b) is concerned with a spatial economy without regional dimension. The paper is organized as follows. Section 2 defines the basic model. Section 3 demonstrates how to solve the equilibrium economic structure with the general production functions and any number of regions and shows the existence of a unique equilibrium with the Cobb-Douglas production function for a 3 regional economy. Section 4 studies the impact of technological change upon the spatial equilibrium. Section 5 demonstrates how changes in the rate of interest, the preference, and the inflation policy affect the entire national economy. Section 6 concludes the study. Appendix proves the main results in Section 3.

2. The multi-region trade model with capital accumulation

In describing economic production, we follow the neoclassical trade framework. The system consists of multiple regions, indexed by $j = 1, ..., J$. It is assumed that the regions produce a homogenous commodity. Regions trade because of differences in household behavior and productivity. We assume that the economy is too small to affect the world interest rate. The households hold wealth and land and receive income from wages and interest payments of wealth. Land is only for residential use. Technologies of all the production sectors are characterized of constant returns to scale. All markets are perfectly competitive and capital and labor are completely mobile between the sectors and capital is perfectly mobile in interregional and international markets. We neglect possible international emigration or/and immigration. Most aspects of production sectors in our model are similar to the neo-classical one-sector growth model. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use capital and labor. Commodities are traded without any barriers such as transport costs or tariffs.

The economy has two assets, domestic money and traded capital goods. The economy consists of consumers, firms and the government. The foreign price of traded goods is given in the world market. The domestic residents may hold two assets, domestic money and capital. We neglect transport cost, customs, or any other possible impediments to trade. Let prices be measured in terms of the commodity and the price of the commodity be unity. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the j th region. The interest rate is identical throughout the national economy, i.e., $r(t) = r^*$, where r^* is the rate of interest fixed in the international economy. We assume a homogenous population. A person is free to choose his residential location. We assume that any person chooses the same region where he works and lives. Each region has fixed land. First, we describe production behavior. We introduce

N — the given population of the economy;

L_j — the given (residential) area of region j;

$K(t)$ and $\bar{K}(t)$ — the capital stocks employed by and the total wealth of the national economy at t;

$F_j(t)$ — the output levels of region j's production sector at time t; and
$K_{ij}(t)$ and $N_j(t)$ — the levels of capital stocks and labor force employed by region j's production sector.

Behavior of producers

We use production functions to describe the physical facts of a given technology. We assume that there are only two productive factors, capital $K_{ij}(t)$ and labor $N_j(t)$, at each point of time t. The production functions are given by

$$F_j\left(K_{ij}(t), N_j(t)\right), \quad j = 1, \cdots, J,$$

where F_j are the output of region j. Assume F_j to be neoclassical. We have

$$f_j(t) = f_j\left(k_j(t)\right), \quad f_j(t) \equiv \frac{F_j(t)}{N_j(t)}, \quad k_j(t) \equiv \frac{K_{ij}(t)}{N_j(t)}. \tag{1}$$

The functions, f_j, have the following properties: (i) $f_j(0) = 0$; (ii) f_j are increasing, strictly concave on R^+, and C^2 on R^{++}; $f_j^{'}(k_j) > 0$ and $f_j^{''}(k) < 0$; and (iii) $\lim_{k_j \to 0} f_j^{'}(k_j) = \infty$ and $\lim_{k_j \to +\infty} f_j^{'}(k_j) = 0$.

Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest, r^*, and wage rates, $w_j(t)$, are determined by markets. For any individual firm, r^* and $w_j(t)$ are given at each point of time. The production sector chooses the two variables, $K_j(t)$ and $N_j(t)$, to maximize its profit. The marginal conditions are given by

$$r^* + \delta_k = f_j^{'}(k_j), \quad w_j(t) = f_j(k_j) - k_j f_j^{'}(k_j), \tag{2}$$

where δ_k is the depreciation rate of physical capital. As r^* is fixed, from $r^* + \delta_k = f_j^{'}$ and $w_j = f_j - k_j f_j^{'}$ we obtain that both $k_j(t)$ and $w_j(t)$ are functions of r^* as

$$k_j = \phi_{kj}(r^*), \quad w_j = \phi_{wj}(r^*). \tag{3}$$

We see that k_j and w_j are invariant in time. It is straightforward to show $d\phi_{kj} / dr^* = 1/ f_j^{''} < 0$ and $d\phi_{wj} / dr^* = -\phi_{kj} < 0$. As the rate of interest rises, both the capital densities and the wage rates fall.

Behavior of consumers

The spatial aspects of this model follow the equilibrium theory of urban land market pioneered by Alonso (1964). This approach has been extended in many directions. The earlier important contributions are made by Muth (1969), Mills (1967), Beckmann (1969), Solow (1973), and others. There are many contributions in the literature in the last two decades. As an extension of Solow and Vickrey (1971) and Beckmann (1976), Ogawa and Fujita (1980) introduced multiple types of interactions. Their model includes possible existence of multiple urban centers. Since then, there are many works on polycentric cities.[5] As the other aspects of our model are already complicated, we will use a simple form of spatial structure. In our dynamic model, we also assume that all residents in a region work in the region's CBD. People travel only between their homes and the CBD. Travel is equally costly in terms of time or/and money in all directions. An individual may reside at only one location. The only spatial characteristic of any location that directly matters is the distance from the city center. The regional system is geographically linear and consists of two parts - the CBD and the residential area. The geography consists of a finite strip of land extending from the CBD with constant unit width. We assume that the production sectors are concentrated in the CBD. The households occupy the residential area. We assume that the CBD is located at the left-side end of the linear territory, as illustrated in Figure 1. As we will get the same conclusions if we locate the CBD at the center of the linear system, the specified urban configuration will not affect our discussion.

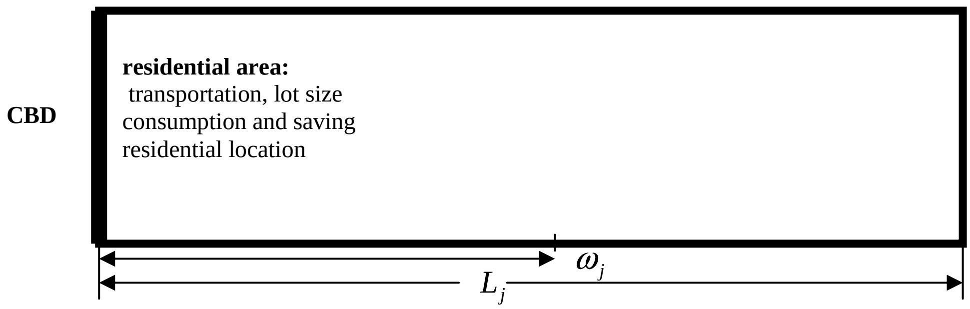


Figure. 1 The Spatial Configuration of a Region

We now describe housing production and behavior of households. First, we introduce

- ω_j — the distance from the CBD to a location in the residential area in region j;
- $R_j(\omega_j, t)$ and $R_{hj}(\omega_j, t)$ — the land rent and housing rent per household at location ω_j;
- $\bar{k}_j(\omega_j, t)$ and $s_j(\omega_j, t)$ — the wealth owned by and the saving made by the household at location ω_j, respectively;
- $c_j(\omega_j, t)$ — the consumption of the household at location ω_j;

[5] See, for instance, Asami et al. (1990), and Papageorgiou and Pines (1999).

- $n_j(\omega_j, t)$ and $L_{hj}(\omega_j, t)$ — the residential density and the lot size of the household at location ω_j; and
- $K_{hj}(t)$ — the capital stocks employed by the housing sector in region j.

We assume that all housing is residential housing. To explain space-dependent gradients for residential density and capital-land ratios, Muth introduces a commodity "housing" rather than land in describing dwelling conditions. Housing is produced with land and non-land inputs. Let us denote $c_{hj}(\omega_j, t)$ housing service received by the household at location ω_j. We specify the housing service production function as follows

$$c_{hj}(\omega_j, t) = A_{hj} k_{hj}^{\alpha_h}(\omega_j, t) L_{hj}^{\beta_h}(\omega_j, t), \ \ \alpha_h + \beta_h = 1, \ \ \alpha_h, \beta_h \geq 0, \quad (4)$$

where $k_{hj}(\omega_j, t)$ is the input level of capital per household at location ω_j. Here, we assume that housing capital can be instantaneously adjusted. All the characteristics of houses such as the size of a lot and the size of a house can be changed instantaneously without costs. Hence the capital-land ratio is always perfectly adjusted.[6] The marginal conditions for the housing sectors are given by

$$r^* + \delta_k = \frac{\alpha_h R_{hj}(\omega_j, t) c_{hj}(\omega_j, t)}{k_{hj}(\omega_j, t)}, \ \ R_j(\omega_j, t) = \frac{\beta_h R_{hj}(\omega_j, t) c_{hj}(\omega_j, t)}{L_{hj}(\omega_j, t)}, \ \ 0 \leq \omega_j \leq L_j. \quad (5)$$

According to the definitions of $L_{hj}(\omega_j, t)$ and $n_j(\omega_j, t)$, we have

$$n_j(\omega_j, t) = \frac{1}{L_{hj}(\omega_j, t)}, \ \ 0 \leq \omega_j \leq L_j. \quad (6)$$

The total capital stocks employed by region $j's$ housing sector is equal to the sum of the capital stocks for housing over space at any point of time. The relationship between $k_{hj}(\omega_j, t)$ and $K_{hj}(t)$ is thus given by

$$K_{hj}(t) = \int_0^{L_j} n_j(\omega_j, t) k_{hj}(\omega_j, t) d\omega_j, \quad j = 1, \cdots, J. \quad (7)$$

We assume that agents have perfect foresight with respect to all future events and capital markets operate frictionless. The government levies no taxes. Money is introduced by assuming that a central bank distributes at no cost to the population a per capita amount of fiat money $M(t) > 0$. The scheme according to which the money stock evolves over time

[6] Our approach is mainly based on Anas (1978), even though the Anas housing production also includes the labor input. Further issues related to the durability of real estates and its costly conversion and replacements are also discussed by Anas. See also Arnott (1980), Arnott et al. (1999) and Glaeser and Gyourko (2005) for introducing more realistic aspects of housing market to the growth model. A recent literature review is referred to Lin et al. (2004).

is deterministic and known to all agents. With μ being the constant net growth rate of the money stock, $M(t)$ evolves over time according: $\dot{M}(t) = \mu M(t)$, $\mu > 0$. At t the government brings $\mu M(t)$ additional units of money per capita into circulation in order to finance all government expenditures via seigniorage. Let $m(t)$ stand for the real value of money per capita measured in units of the output good, that is, $m(t) = M(t)/P(t)$. The government expenditure in real terms per capita, $\tau(t)$, is given by

$$\tau(t) = \frac{\dot{M}(t)}{P(t)} = \frac{\mu M(t)}{P(t)} = \mu m(t). \tag{8}$$

The representative household receives $\mu m(t)$ units of paper money from the government through a "helicopter drop", also considered to be independent of his money holdings. Let $\overline{m}_j(\omega, t)$ stand for the real money holdings of the typical household at ω in region j. According to the definitions of $m(t)$, $\overline{m}_j(\omega, t)$ and $n_j(t)$, we have

$$\sum_{j=1}^{J} \int_0^{L_j} \overline{m}_j(\omega_j, t) n_j(\omega_j, t) d\omega_j = N m(t). \tag{9}$$

Each worker may get income from land ownership, wealth ownership and wages. Let $\bar{k}_j(\omega_j, t)$ stand for the per capita wealth (excluding land) owned by the typical household at location ω_j and $y_j(t, \omega)$ for the real physical wealth owned and real current income by the representative household in region j. Then we have

$$y_j(\omega_j, t) = r^* \bar{k}_j(\omega_j, t) + w_j(t) + \mu m(t), \quad 0 \le \omega \le L_j .$$

We call y_j the current income in the sense that it comes from the wage, the earnings from ownership of wealth, and the money distribution from the government. The real disposable income of the representative household in region j, $\hat{y}_j(t)$, is given as the sum of the real current income and wealth in real terms, defined as follows

$$\hat{y}_j(\omega_j, t) = y_j(\omega_j, t) + a_j(\omega_j, t) - \pi(t)\overline{m}_j(\omega_j, t), \tag{10}$$

where $a_j(\omega_j, t)$ is the real total wealth in domestic currency defined as

$$a_j(\omega_j, t) \equiv \overline{m}_j(\omega_j, t) + \bar{k}_j(\omega_j, t).$$

It should be noted that the variable, $\bar{k}_j(t)$, in the definition of the current income is considered as a flow variable, just like the income, $y_j(t)$. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we may consider

$\bar{k}_j(t)$ as the amount of the "income" that the consumer obtains at time t by selling all of his wealth. Hence, at time t the consumer has the total amount of income equaling $\hat{y}_j(t)$ to distribute among holding money, consuming and saving. It should also be remarked that in the growth literature, for instance, in the Solow model, the saving is out of the current income, $y_j(t)$, while in this study the saving is out of the disposable income.

At each point of time, a consumer at location ω_j distributes the total available budget among the leisure time, $T_{hj}(\omega_j, t)$, housing, $c_{hj}(\omega_j, t)$, saving, $s_j(\omega_j, t)$, consumption of industrial goods, $c_j(\omega_j, t)$. Here, we assume that the leisure is only dependent on the residential location as the work time is fixed and equal for each household, in disregard of residential location. After the work time is decided, the households decide the time distribution between leisure and travel to work. As we assume that the travel time from the CBD to the residential location is only related to the distance and neglect any other effects such on technological change, infrastructure improvement, and congestion on the travel time form the CBD to the residential area, the leisure time, which is equal to the fixed total time minus the travel time, is only related to location. Let T_0 and $\Gamma_j(\omega_j, t)$ respectively stand for the total available time for travel and leisure and the time spent on traveling between the residence and CBD. We have

$$T_{hj}(\omega_j, t) = T_0 - \Gamma_j(\omega_j, t).$$

The budget constraint is given by

$$(1 + r^*)\bar{m}_j(\omega_j, t) + R_{hj}(\omega_j, t)c_{hj}(\omega_j, t) + c_j(\omega_j, t) + s_j(\omega_j, t) = \hat{y}_j(\omega_j, t) - \Gamma_{jc}(\omega_j, t),$$
$$0 \le \omega_j \le L_j, \quad j = 1, \cdots, J. \tag{11}$$

Equation (11) means that the consumption and saving exhaust the consumers' disposable personal income. Substituting $T_{hj}(\omega_j, t) = T_0 - \Gamma_j(\omega_j, t)$ and the definition of $\hat{y}_j(t, \omega_j)$ into (11), we have

$$(\pi(t) + r^*)\bar{m}_j(\omega_j, t) + R_{hj}(\omega_j, t)c_{hj}(\omega_j, t) + c_j(\omega_j, t) + s_j(\omega_j, t) = \bar{y}_j(\omega_j, t), \tag{12}$$

where

$$\bar{y}_j(\omega_j, t) \equiv (1 + r^*)\bar{k}_j(\omega_j, t) + w_j(t) + \mu m(t) - \Gamma_{jc}(\omega_j, t).$$

Location choice is closely related to the existence and quality of such physical environmental attributes as open space and noise pollution as well as social environmental quality. We assume that utility level, $U(\omega, t)$, of the household at location ω_j is dependent on $T_{hj}(\omega_j, t)$, $\bar{m}_j(\omega_j, t)$, $c_{hj}(\omega_j, t)$, $s_j(\omega_j, t)$, and $c_j(\omega_j, t)$ as follows

$$U_j(\omega_j, t) = \theta_j(\omega_j, t)T_{hj}^{\sigma}(\omega_j, t)\bar{m}_j^{\varepsilon_0}(\omega_j, t)c_j^{\xi_0}(\omega_j, t)c_{hj}^{\eta_0}(\omega_j, t)s_j^{\lambda_0}(\omega_j, t), \quad \sigma, \varepsilon_0, \xi_0, \eta_0, \lambda_0 > 0, \tag{13}$$

in which σ, ε_0, ξ_0, η_0, and λ_0 are a typical person's elasticity of utility with regard to leisure time, holding money, industrial goods, housing, and saving at ω_j. We call σ, ε_0, ξ_0, η_0, and λ_0 propensities to use leisure time, to hold real money, to consume goods, to consume housing, and to hold wealth, respectively.

Maximizing $U_j(\omega_j, t)$ subject to the budget constraint (10) yields

$$c_j(\omega_j, t) = \xi \bar{y}_j(\omega_j, t), \;\; \bar{m}_j(\omega_j, t) = \frac{\varepsilon \bar{y}_j(\omega_j, t)}{\pi(t) + r^*}, \;\; c_{hj}(\omega_j, t) = \frac{\eta \bar{y}_j(\omega_j, t)}{R_{hj}(\omega_j, t)}, \;\; s_j(\omega_j, t) = \lambda \bar{y}_j(\omega_j, t), \tag{14}$$

where

$$\xi \equiv \rho \xi_0, \;\; \varepsilon \equiv \rho \varepsilon_0, \;\; \eta \equiv \rho \eta_0, \;\; \lambda \equiv \rho \lambda_0, \;\; \rho \equiv \frac{1}{\xi_0 + \varepsilon_0 + \eta_0 + \lambda_0}.$$

The above equations mean that the housing consumption, consumption of the good and saving are positively proportional to the available income.

In this study, we incorporate amenity into the consumer location decision by assuming that amenity is a function of residential density.[7] Distance from the CBD reflects two elements: the inconvenience of the distance and the value of the amenity of the surrounding area. This study considers that residential densities may have positive or negative agglomeration effects. We specify the amenity, $\theta_j(\omega_j, t)$, at ω_j as follows

$$\theta_j(\omega_j, t) = \theta_{0j} n_j^{\bar{\mu}}(\omega_j, t), \;\; \theta_{0j} > 0. \tag{15}$$

The function, $\theta_j(\omega_j, t)$, implies that the amenity level at location ω_j is related to the residential density at the location.[8] It is assumed that all households obtain the same level of utility at any point of time. This also comes out of our assumption that the population is homogeneous and people can change their residential location freely without any transaction costs and time delay.[9] The conditions that households get the same level of utility at any location at each point of time is represented by

$$U_j(\omega_1, t) = U_m(\omega_m, t), \;\; j, m = 1, \cdots, J, \;\; 0 \le \omega_1 \le L_j, \;\; 0 \le \omega_m \le L_m. \tag{16}$$

[7] This study does not take account of externalities for producers. For instance, firms often prefer to locate to other firms. An explicit introduction of externalities will make the spatial structure far more complicated. In the literature of urban economics, various externalities have been analyzed (see, Henderson, 1974, Upton, 1981, and Abdel-Rahman, 2004).

[8] This specified form is a limited case. Locational amenities or disamenities are not only affected by the residential density at the location. For instance, possible social contacts of any individual are spread over the whole space. Air pollution is not limited to locals.

[9] In reality, to change housing location costs. Although the condition of utility equalization is often used in the literature of urban economics, the assumption of utility equalization is rarely used in the literature of economic dynamics as the temporary equilibrium condition of population distribution.

According to the definition of $s_j(\omega_j, t)$, the capital accumulation for the household at location ω_j is given by

$$\dot{a}_j(\omega_j, t) = s_j(\omega_j, t) - a_j(\omega_j, t), \quad 0 \le \omega_j \le L_j. \tag{17}$$

Regional population and capital

A region's population is distributed over the whole urban area. Region $j's$ population constraint is given by

$$\int_0^{L_j} n_j(\omega_j, t) d\omega_j = N_j(t), \quad j = 1, \cdots, J. \tag{18}$$

The above equation also implies that each region's labor force is fully employed. Region $j's$ consumption, $C_j(t)$, is given by

$$\int_0^{L_j} c_j(\omega_j, t) n_j(\omega_j, t) d\omega_j = C_j(t). \tag{19}$$

Region $j's$ saving is given by

$$\int_0^{L_j} s_j(\omega_j, t) n_j(\omega_j, t) d\omega_j = S_j(t). \tag{20}$$

The total capital stock employed by region j is the sum of the capital stocks employed by region $j's$ industrial and housing sectors

$$K_{ij}(t) + K_{hj}(t) = K_j(t). \tag{21}$$

The total wealth owned by region $j's$ households is given by

$$\int_0^{L_j} \bar{k}_j(\omega_j, t) n_j(\omega_j, t) d\omega_j = \bar{K}_j(t), \quad j = 1, \cdots, J. \tag{22}$$

National population and capital

The total capital stocks employed by the economy is equal to the sum of the capital stocks employed by the capital stocks employed by all the regions. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \left(K_{ij}(t) + K_{hj}(t)\right). \tag{23}$$

The total wealth of the national economy is the sum of the wealth owned by all the households

$$\bar{K}(t) = \sum_{j=1}^{J} \bar{K}_j(t). \tag{24}$$

We introduce $B(t)$ as the value of the economy's net foreign assets at t. The income from the net foreign assets, $E(t)$, which may be either positive, zero, or negative, is equal to $r^* B(t)$. The assumption that the national labor force is fully employed is represented by

$$\sum_{j=1}^{J} N_j(t) = N. \tag{25}$$

We now explain trade balance in the model. According to the definitions of the national wealth, the capital stocks employed by the economy and the net foreign assets, we have

$$\bar{K}(t) = K(t) + B(t).$$

We have thus built the multi-regional model of a small open economy with multiple regions and capital accumulation.

3. Spatial Equilibrium and Existence of Equilibrium by Simulation

The previous section develops a general model of the national economy with multiple regions and residential distribution. The model is structurally general in the sense that by adding some assumptions, the model can be applied to describe national as well as international economies. For instance, if we allow all the regions have the identical conditions and neglect the role of money and housing, then the model becomes the one-sector growth model of national economy. Although the behavioral mechanism of consumers of our model is different from the Solow model, the model with the identical regions and without money and housing sector is mathematically identical with the Solow model – it has a unique stable equilibrium. It can be seen that the national model with housing and without money is a two-sector growth model. If we don't allow people to be interregionally mobile and neglect the role of money, then our model is mathematically identical with the Oniki-Uzawa trade model with constant saving rates. It should be noted that the Oniki-Uzawa trade model and most of its extensions are limited to two national economies and our analytical results are valid for any number of regions. The residential distribution in our model is according to the standard Alonso model and the housing sector is influenced by Muth's housing model. Although our model is structurally general, it is important to know whether it is analytically tractable. In this paper we are concerned with equilibrium, because a genuine dynamic analysis is too complicated. We now show how to find equilibrium in our model. The following lemma, which provides a procedure for determining the equilibrium values of all the variables, is proved in Appendix A1.

Lemma 1

For given rate of interest, r^*, k_j and w_j, $j = 1, 2, \cdots, J$, are determined by equations (2). The real money per capita, m, is determined by the following equation

$$\sum_{j=1}^{J} N_j \left\{ r_j - \frac{r_0 \phi_j}{\bar{k}_j^{\beta_k}(0) T_{hj}^{\beta_0}(0)} \int_0^{L_j} \Gamma_{jc}(\omega_j) \bar{k}_j^{\beta_k}(\omega_j) T_{hj}^{\beta_0}(\omega_j)\, d\omega_j \right\} = \left(\frac{\bar{\lambda}}{\varepsilon_\pi} - \mu_0 \right) Nm, \qquad (26)$$

where

$$\bar{k}_j(\omega_j) = r_j + \mu_0 m - r_0 \Gamma_{jc}(\omega_j), \;\; 0 \le \omega_j \le L_j, \qquad (27)$$

in which $\bar{\lambda} \equiv \lambda - \varepsilon_\pi$, $r_j \equiv r_0 w_j$, $\mu_0 \equiv r_0 \mu$, and $r_0 \equiv 1/(1/\bar{\lambda} - 1 - r^*)$, and the labor distribution, N_j, are functions of $\bar{k}_j(\omega_j)$ determined as follows

$$N_j = \frac{\bar{\phi}_j N}{\sum_{j=1}^{J} \bar{\phi}_j}, \quad j = 1, \cdots, J,$$

where $\bar{\phi}_1 = 1$ and

$$\bar{\phi}_j \equiv \frac{\phi_1}{\phi_j} \left(\frac{\bar{k}_j^{1/\rho - \beta_h \eta_0}(0) \theta_{0j} T_{hj}^{\sigma}(0) A_{hj}^{\eta_0}}{\bar{k}_1^{1/\rho - \beta_h \eta_0}(0) \theta_{01} T_{h1}^{\sigma}(0) A_{h1}^{\eta_0}} \right)^{1/(\beta_h \eta_0 - \bar{\mu})}, \quad j = 2, \ldots J.$$

The equilibrium values of all the other variables are given by the following procedure: f_j by (1) → $F_j = f_j N_j$ → $K_{ij} = k_j N_j$ → $\bar{k}_j(\omega_j)$ by (27) → $\bar{y}_j(\omega_j) = \bar{k}_j(\omega_j)/\lambda$ → $n_j(\omega_j)$ by (A11) and (A12) → $L_{hj}(\omega_j) = 1/n_j(\omega_j)$ → $\bar{K}_j = \int_0^{L_j} \bar{k}_j(\omega_j) n(\omega_j) d\omega_j$ → $\bar{Y}_j$ by (A4) → $k_{hj}(\omega_j)$ by (A6) → K_{hj} by (A7) → $K_j = K_{ij} + K_{hj}$ → $R_{hj}(\omega_j)$ by (A9) → $R_j(\omega_j)$ by (5) → $c_j(\omega_j)$, $\bar{m}_j(\omega_j)$, $c_{hj}(\omega_j)$ and $s_j(\omega_j)$ by (14).

For illustration, we will follow the procedure given in Lemma 1 to solve the equilibrium values of the system. In the remainder of this study, we are concerned with an economy with 3 regions.[10] We specify the production functions as follows

$$F_j(t) = A_j K_{ij}^{\alpha_j}(t) N_j^{\beta_j}(t), \;\; \alpha_j + \beta_j = 1, \;\; \alpha_j, \beta_j > 0, \quad j = 1, 2, 3,$$

where A_j is region j's productivity. From $f_j = A_j k_j^{\alpha_j}$, and equations (2), we have

[10] As the procedure in Lemma 1 is valid for any number of regions, one can similarly examine the behavior for the system with any number of regions.

$$k_1 = \left(\frac{\alpha_1 A_1}{r^* + \delta_k}\right)^{1/\beta_1}, \quad k_j = \phi_{kj}(k_1) \equiv \left(\frac{\alpha_1 A_1 k_1^{-\beta_1}}{\alpha_j A_j}\right)^{-1/\beta_j}, \quad j = 2, 3.$$

By equations (3), we have

$$\phi_{wj} = A_j \beta_j \phi_{kj}^{\alpha_j}(k_1), \; \hat{y}_j = \phi_r(k_1)\bar{k}_j + A_j \beta_j \phi_j^{\alpha_j}(k_1), \quad j = 1, 2, 3, \qquad (28)$$

in which we use $\phi_{k1}(k_1) = k_1$ and $\phi_r(k_1) \equiv \alpha_1 A_1 k_1^{-\beta_1} + \delta$ and $\delta \equiv 1 - \delta_k$.

We specify the travel time and cost functions as follows

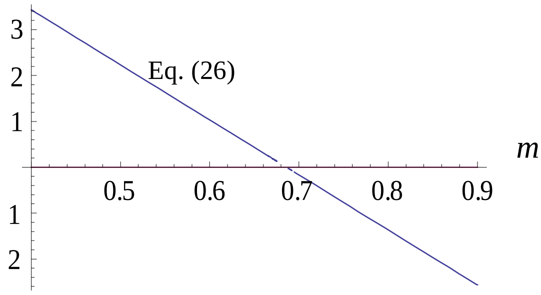
$$\Gamma_j(\omega_j) = \nu_j \omega_j, \; 0 \le \omega_j \le L_j, \; \Gamma_{cj}(\omega_j) = \nu_{cj}\omega_j, \; 0 \le \omega_j \le L_j. \qquad (29)$$

The inverse of the parameter, ν_j, is the travel speed in region j. Hence, a decrease in the value of ν_j is interpreted as an improvement in region j's transportation system. The parameter, ν_{cj}, is the travel cost per unit of distance in region j. The transportation systems in this study are measured by the parameters, ν_j and ν_{cj}. Following Lemma 1, we can determine equilibrium of the system with the parameter values specified as follows $r^* = 0.06$, $N = 10$, $T_0 = 1$, $\mu = 0.03$, $\delta_k = 0.05$, $\alpha_j = 0.3$, $\alpha_h = 0.4$, $\bar{\mu} = -0.1$, $\sigma = 0.2$, $\varepsilon_0 = 0.03$, $\eta_0 = 0.08$, $\xi_0 = 0.14$, $\lambda_0 = 0.74$,

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 1.1 \\ 1.05 \\ 1 \end{pmatrix}, \begin{pmatrix} A_{h1} \\ A_{h2} \\ A_{h3} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.7 \\ 0.7 \end{pmatrix}, \begin{pmatrix} \theta_{01} \\ \theta_{02} \\ \theta_{03} \end{pmatrix} = \begin{pmatrix} 4.2 \\ 4.2 \\ 4.4 \end{pmatrix}, \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} 0.02 \\ 0.03 \\ 0.04 \end{pmatrix}, \begin{pmatrix} \nu_{c1} \\ \nu_{c2} \\ \nu_{c3} \end{pmatrix} = \begin{pmatrix} 0.07 \\ 0.05 \\ 0.05 \end{pmatrix}, \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}, \qquad (30)$$

The rate of interest is fixed at 6 percent in the international market. Region 1 has the highest level of productivity. Region 2 's level of productivity is the second, next to region 1's. We term region 1 as the advanced region, region 2 the middle region, and region 3 the remote region. Although the specified values are not based on empirical observations, the choice does not seem to be unrealistic. For instance, some empirical studies on the US economy demonstrate that the value of the parameter, α, in the Cobb-Douglas production is approximately equal to 0.3 (for instance, Abel and Bernanke, 1998). With regard to the technological parameters, what are important in our interregional study are their relative values. The presumed productivity differences among the regions are not very large. This is similarly true for the specified differences in the amenity parameters among regions. Region 1's land area is smaller than region 2's and region 3's land area is equal to that of region 1. The amenity parameters of the three regions are equal. Region 1's amenity parameter is the same as region 2's and region 3's amenity parameter is highest. The three regions have different transportation conditions. preference specification implies that the consumer spends about 8 per cent of the available income on housing and about 17 percent on the other expenditures.

Following Lemma 1, we determine the equilibrium values of the variables. From equation (26), we determine the value of the real money. It is further demonstrated that the equation has a unique solution.



Following Lemma 1, we solve the other variables and list the simulation results for the location-independent variables as in (31)

$$F = 18.713,\ F_i = 17.066,\ F_h = 1.647,\ K = 52.531,\ \overline{K} = 8.370,\ E = -2.650,\ m = 0.686,$$

$$\begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 4.904 \\ 4.495 \\ 4.193 \end{pmatrix},\ \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} 1.762 \\ 1.648 \\ 1.537 \end{pmatrix},\ \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1.233 \\ 1.154 \\ 1.076 \end{pmatrix},\ \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 6.423 \\ 2.284 \\ 1.293 \end{pmatrix},\ \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 11.314 \\ 3.765 \\ 1.987 \end{pmatrix},$$

$$\begin{pmatrix} F_{h1} \\ F_{h2} \\ F_{h3} \end{pmatrix} = \begin{pmatrix} 1.097 \\ 0.358 \\ 0.192 \end{pmatrix},\ \begin{pmatrix} K_{i1} \\ K_{i2} \\ K_{i3} \end{pmatrix} = \begin{pmatrix} 30.856 \\ 10.268 \\ 5.420 \end{pmatrix},\ \begin{pmatrix} \overline{K}_1 \\ \overline{K}_2 \\ \overline{K}_3 \end{pmatrix} = \begin{pmatrix} 5.575 \\ 1.822 \\ 0.974 \end{pmatrix},\ \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} 34,843 \\ 11.571 \\ 6.117 \end{pmatrix},\ \begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} = \begin{pmatrix} 4.569 \\ 1.493 \\ 0.798 \end{pmatrix}, \quad (31)$$

in which $E \equiv r^*\left(\overline{K} - K\right)$ and

$$F \equiv \sum_{j=1}^{3}\left(F_j + F_{hj}\right),\ F_i \equiv \sum_{j=1}^{3} F_j,\ F_h \equiv \sum_{j=1}^{3} F_{hj},\ F_{hj} \equiv \int_0^{L_j} R_{hj}\, c_{hj}\, n_j\, d\omega_j,\ M_j \equiv \int_0^{L_j} \overline{m}_j\, n_j\, d\omega_j,$$

The variable, E, measures trade balance of the national economy. If $E > (<)\ 0$, the country is said to be in trade surplus (deficit); if $E = 0$, the country is said to be in trade balance. We don't represent E_j (which are the regions' trade balances and are given by $r^*\left(\overline{K}_j - K_j\right)$). The national domestic product, F, is equal to the sum of the national output levels of the industrial and housing sectors of all the regions, F_i and F_h.

We see that the capital intensity, per capita output level, and wage rate in the advanced region are higher than the corresponding variables in the middle region; these variables in the middle region are higher than the corresponding variables in the remote region. Most of the population and national product are concentrated in the advanced region. The advanced region attracts almost 64 of the national population. The national economy is in trade deficit. The level of the capital stocks employed by the economy is much higher than the wealth owned by the economy. The advanced region produces more industrial and housing outputs than the other two regions and it employs more capital stocks than the other two regions. The location-dependent variables are plotted in Figure 2. From the figure, we see that the residential density, consumption level of industrial goods, the land rent and housing rent each region fall in distance. Although workers can earn higher wages in the advanced region than in the remote region, they pay much higher rent for housing than the households in the remote region. Expectably, the typical household in the advanced region consumes more goods and lives in a smaller house than the typical household in the remote region. As

$$s_j(\omega_j) = \frac{\lambda \eta c_j(\omega_j)}{\xi}, \quad R_{hj}(\omega_j) c_{hj}(\omega_j) = \frac{\eta c_j(\omega_j)}{\xi},$$

we see that $s_j(\omega)$ and $R_{hj}(\omega_j) c_{hj}(\omega_j)$ vary in distance proportionally to $c_j(\omega_j)$ at any location. Hence, we do not present $s_j(\omega)$ and $R_{hj}(\omega_j) c_{hj}(\omega_j)$. The variables for each region change in the same way as the region's consumption level of the industrial goods.

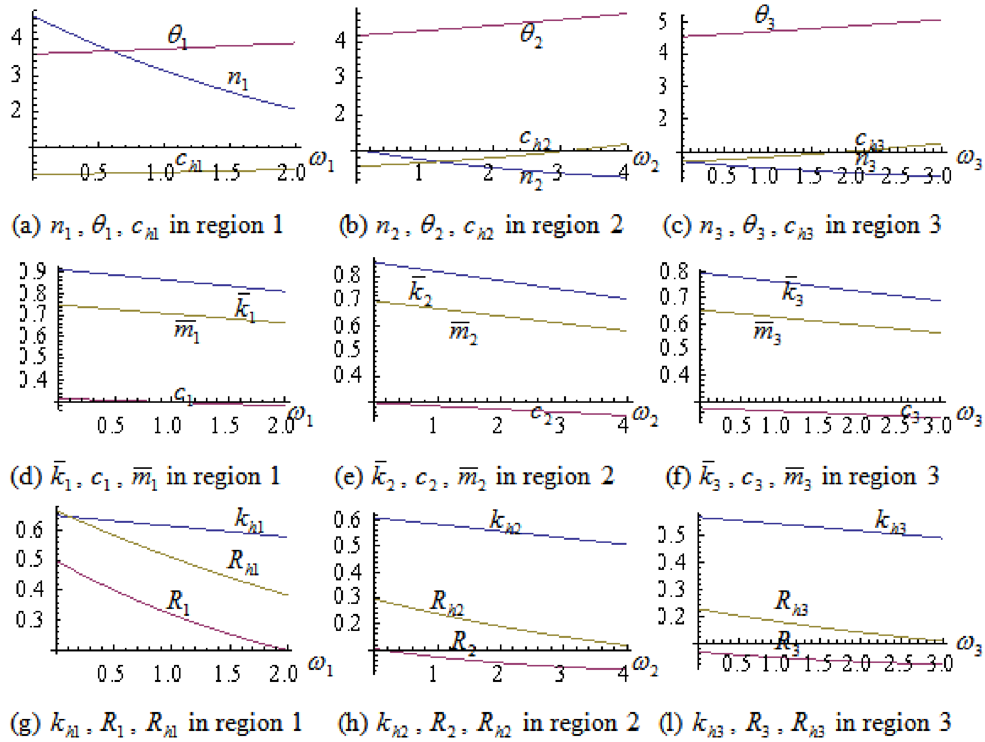


Figure 2. Regional Residential Distribution, Consumption and Rents over Space

4. The Inflation Policy and Regional Technological Changes

This section examines impact of changes in the inflation policy and regional technology on the national economy and regional economic structures. As the system has a unique equilibrium, we can examine the shift of equilibrium point as a parameter is changed. Modern analysis of the long-term interaction of inflation and capital formation begins with

Tobin's seminal contribution (Tobin, 1965).[11] In Tobin's approach, a monetary economy has a real sector exactly like that in the Solow growth model, so that the monetary nature of the model depends on how money is introduced into the model. As a depositor of purchasing power money can be held by private agents as an alternative form of wealth to physical capital stock. Tobin showed that an increase in the level of the inflation rate will increase the capital stock of an economy. As our model is interregional, we can analyze regional effects of the inflation policy.

First, we examine the case that all the parameters, except the inflation policy, μ, are the same as in (30). We increase μ from 0.03 to 0.05. The simulation results are summarized in (32), in which a variable $\bar{\Delta} x_j$ stand for the change rate of the equilibrium value of the variable x_j in percentage due to the change in the parameter value from μ_0 ($= 0.03$ in this case) to μ_1 ($= 0.05$). That is

$$\bar{\Delta} x_j \equiv \frac{x_j(\mu_1) - x_j(\mu_0)}{x_j(\mu_0)} \times 100,$$

where $x_j(\mu)$ stands for the equilibrium value of variable x_j with the parameter value μ. We will use the symbol $\bar{\Delta}$ with the same meaning when we analyze other parameters. We see that as the inflation rate is increased, the national output and capital stocks are increased. This is similar to what the Tobin model predicts. Nevertheless, as our production system consists of two sectors, the impact on the industrial sector is negative but on the housing output is positive. As it is more expensive to hold money, per capita real money is reduced. As the economy is open and the wage rates and capital intensities are determined by technology and the globally fixed rate of interest, the change in the inflation has no impact on the wage rates and the capital intensities. The advanced region becomes less attractive as the inflation rate becomes higher. The population of the advanced region falls and the population levels of the other two regions rise. The change directions in the regional industrial output levels and capital stocks employed by the industrial sectors have the same change rates as the regional population.

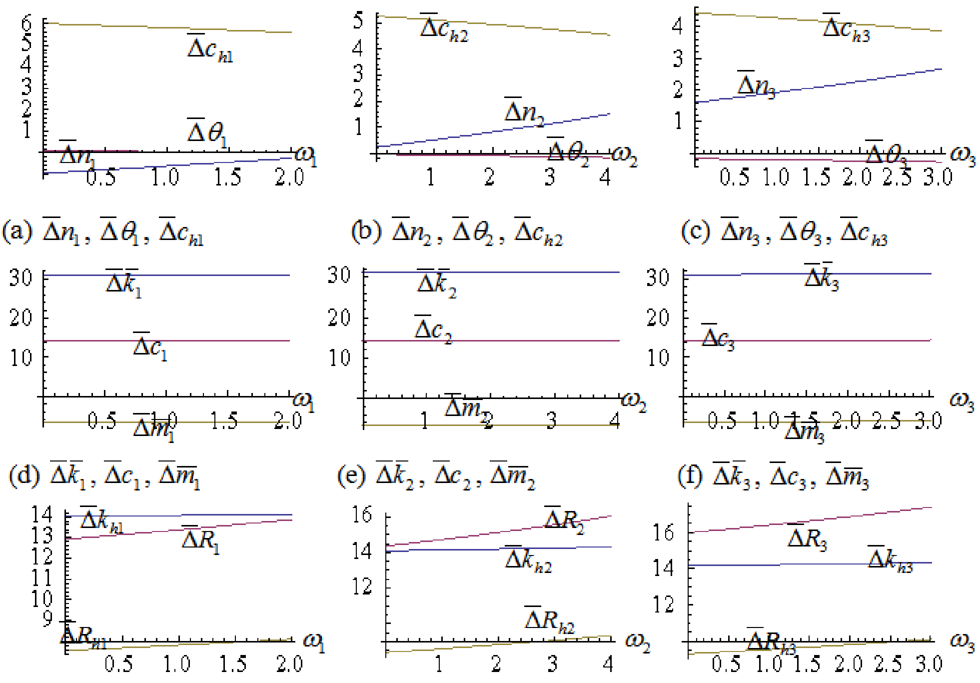
$$\mu : 0.03 \Rightarrow 0.05 \Rightarrow \ \bar{\Delta} F = 1.20, \ \bar{\Delta} F_i = -0.05, \ \bar{\Delta} F_h = 14.02, \ \bar{\Delta} K = 1.56, \ \bar{\Delta} \bar{K} = 31.04,$$
$$\bar{\Delta} E = -4.03, \ \bar{\Delta} m = -6.69,$$

$$\begin{pmatrix} \bar{\Delta} k_1 \\ \bar{\Delta} k_2 \\ \bar{\Delta} k_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta} f_1 \\ \bar{\Delta} f_2 \\ \bar{\Delta} f_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} N_1 \\ \bar{\Delta} N_2 \\ \bar{\Delta} N_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta} F_1 \\ \bar{\Delta} F_2 \\ \bar{\Delta} F_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta} K_{i1} \\ \bar{\Delta} K_{i2} \\ \bar{\Delta} K_{i3} \end{pmatrix} = \begin{pmatrix} -0.66 \\ 0.70 \\ 2.01 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} \bar{K}_1 \\ \bar{\Delta} \bar{K}_2 \\ \bar{\Delta} \bar{K}_3 \end{pmatrix} = \begin{pmatrix} 30.20 \\ 32.08 \\ 33.88 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} 0.94 \\ 2.31 \\ 3.66 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} F_{h1} \\ \bar{\Delta} F_{h2} \\ \bar{\Delta} F_{h3} \end{pmatrix} = \begin{pmatrix} 13.31 \\ 14.95 \\ 16.52 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} M_1 \\ \bar{\Delta} M_2 \\ \bar{\Delta} M_3 \end{pmatrix} = \begin{pmatrix} -7.29 \\ -5.92 \\ -4.67 \end{pmatrix}. \tag{32}$$

[11] See also McCallum (1983) and Walsh (2003) for general discussions on money.

Equation (32) shows the changes of the location-independent variables. The effects on the location-dependent variables are given in Figure 3. We see that the amenities are slightly affected. The consumption levels of goods and housing are increased. Land and housing rents are also increased.



(a) $\bar{\Delta} n_1$, $\bar{\Delta}\theta_1$, $\bar{\Delta} c_{h1}$ (b) $\bar{\Delta} n_2$, $\bar{\Delta}\theta_2$, $\bar{\Delta} c_{h2}$ (c) $\bar{\Delta} n_3$, $\bar{\Delta}\theta_3$, $\bar{\Delta} c_{h3}$

(d) $\bar{\Delta} \bar{k}_1$, $\bar{\Delta} c_1$, $\bar{\Delta} \bar{m}_1$ (e) $\bar{\Delta} \bar{k}_2$, $\bar{\Delta} c_2$, $\bar{\Delta} \bar{m}_2$ (f) $\bar{\Delta} \bar{k}_3$, $\bar{\Delta} c_3$, $\bar{\Delta} \bar{m}_3$

(g) $\bar{\Delta} k_{h1}$, $\bar{\Delta} R_1$, $\bar{\Delta} R_{h1}$ (h) $\bar{\Delta} k_{h2}$, $\bar{\Delta} R_2$, $\bar{\Delta} R_{h2}$ (l) $\bar{\Delta} k_{h3}$, $\bar{\Delta} R_3$, $\bar{\Delta} R_{h3}$

Figure 3. The Spatial Effects of the Inflation Policy

We now increase the productivity level, A_1, from 1.1 to 1.15. The simulation results are summarized in (33) and Figure 4. As the advanced region's productivity is improved, the national output level, F, and total capital, K, the national industrial output level, F_i, and the national housing output level, F_h, are increased. The national wealth is increased and the trade balance is improved. The per capita output level and capital intensity of the advanced region, f_1 and k_1, are increased; the per capita output levels and capital intensities of the other two regions, f_j and k_j, $j = 2, 3$, are not affected. The labor force and output levels of the two sectors in the advanced region, N_1, F_1 and F_{h1}, are increased; the corresponding variables of the other two regions are reduced. The per capita wealth, per capita consumption level and real money holding of any consumer in the three region are increased. As the advanced region increases its productivity, as shown in Figure 4, the land and housing rents in the advanced region rise and the rents in the other two regions fall. The residential density in the advanced region rises and per capita housing consumption level falls; the residential densities in the two other regions fall and per capita housing consumption levels rise.

$$\bar{\Delta}F = 7.87,\ \bar{\Delta}F_i = 7.81,\ \bar{\Delta}F_h = \bar{\Delta}\bar{K} = 8.44,\ \bar{\Delta}K = 7.88,\ \bar{\Delta}E = 7.78,\ \bar{\Delta}m = 8.44,$$

$$\begin{pmatrix} \bar{\Delta}k_1 \\ \bar{\Delta}k_2 \\ \bar{\Delta}k_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta}f_1 \\ \bar{\Delta}f_2 \\ \bar{\Delta}f_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta}w_1 \\ \bar{\Delta}w_2 \\ \bar{\Delta}w_3 \end{pmatrix} = \begin{pmatrix} 6.56 \\ 0 \\ 0 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta}N_1 \\ \bar{\Delta}N_2 \\ \bar{\Delta}N_3 \end{pmatrix} = \begin{pmatrix} 34.23 \\ -61.49 \\ -61.41 \end{pmatrix},\ \begin{pmatrix} \bar{\Delta}F_1 \\ \bar{\Delta}F_2 \\ \bar{\Delta}F_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta}K_{i1} \\ \bar{\Delta}K_{i2} \\ \bar{\Delta}K_{i3} \end{pmatrix} = \begin{pmatrix} 43.03 \\ -61.49 \\ -61.41 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}F_{h1} \\ \bar{\Delta}F_{h2} \\ \bar{\Delta}F_{h3} \end{pmatrix} = \begin{pmatrix} \bar{\Delta}\bar{K}_1 \\ \bar{\Delta}\bar{K}_2 \\ \bar{\Delta}\bar{K}_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta}K_1 \\ \bar{\Delta}K_2 \\ \bar{\Delta}K_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta}M_1 \\ \bar{\Delta}M_2 \\ \bar{\Delta}M_3 \end{pmatrix} = \begin{pmatrix} 43.46 \\ -61.43 \\ -61.35 \end{pmatrix}. \qquad (33)$$

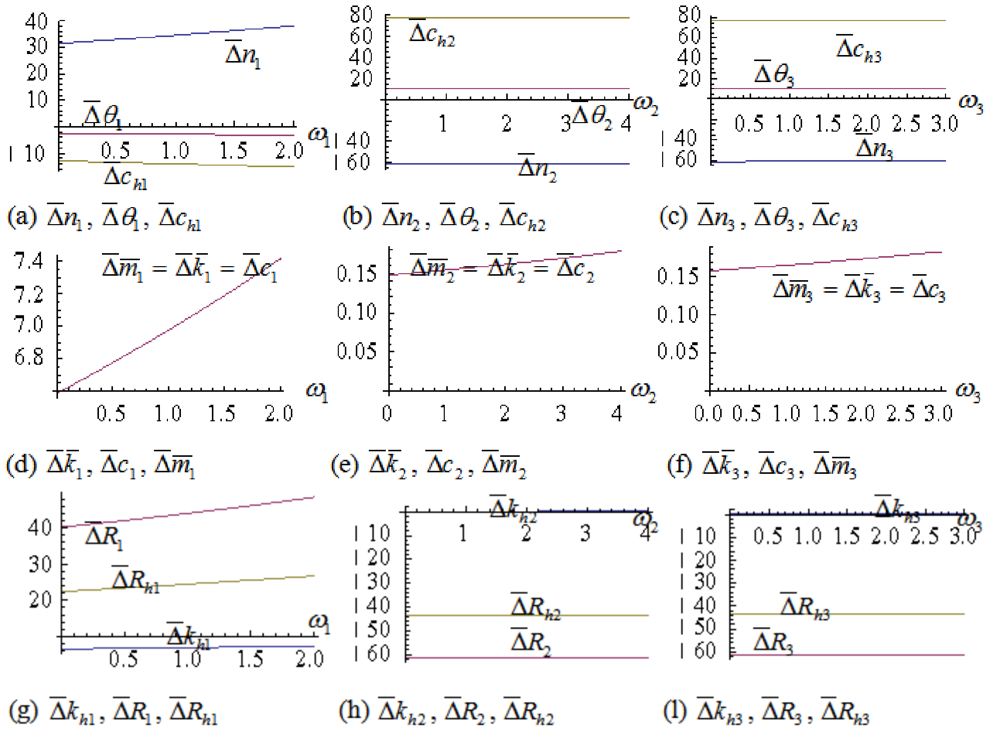


Figure 4. The Spatial Effects of Technological Change in the Advanced Region

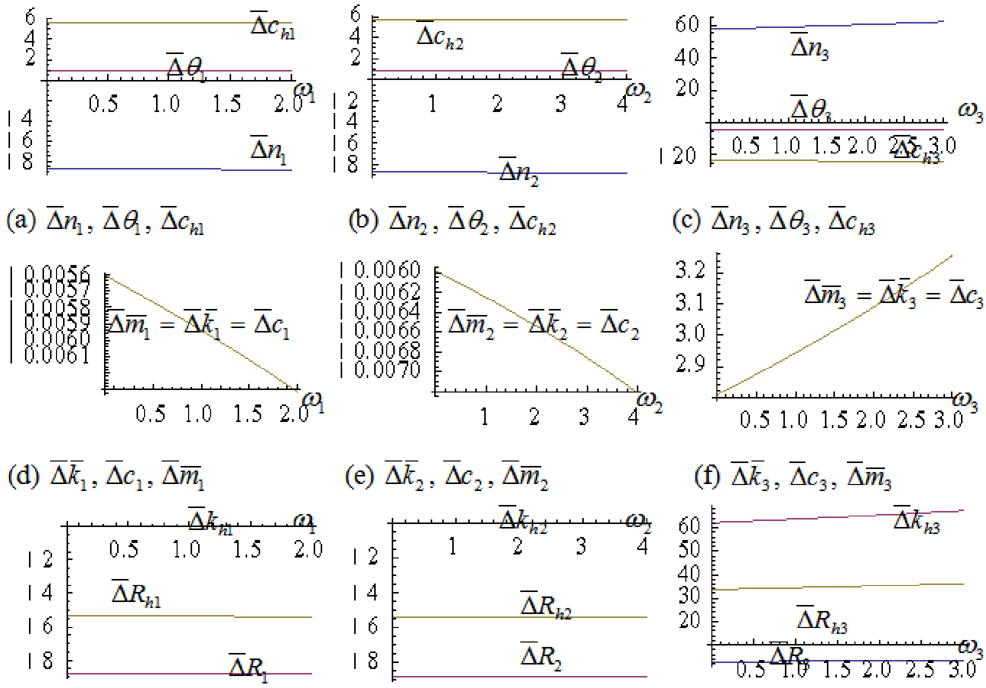
We now allow the remote region's productivity to be increased, with all the other parameter values fixed. We increase A_3 from 1 to 1.02. As the remote region's productivity is improved, the national output level, F, and total capital, K, the national industrial output level, F_i, and the national housing output level, F_h, are all reduced. The effects of the remote region's technological improvement on the national economy are different from the effects of the advanced region's technological improvement. The mechanism for this difference is that as the remote region's productivity is improved, the marginal product of the region's labor will be increased. As the equilibrium is disturbed, some people from the advanced region will migrate to the remote region. As the productivity of the remote region is lower than the advanced region, the net effect is that the national economy produces less as its remote region's productivity is improved.

$$\bar{\Delta} F = \bar{\Delta} F_i = \bar{\Delta} F_h = \bar{\Delta} K = \bar{\Delta} \bar{K} = \bar{\Delta} E = \bar{\Delta} m = -0.34,$$

$$\bar{\Delta} n_j = \bar{\Delta} \theta_j = \bar{\Delta} c_{hj} = \bar{\Delta} \bar{k}_j = \bar{\Delta} k_{hj} = \bar{\Delta} c_j = \bar{\Delta} \bar{m}_j = \bar{\Delta} R_{hj} = \bar{\Delta} R_j = 0, \quad j = 1, 2,$$

$$\begin{pmatrix} \bar{\Delta} k_1 \\ \bar{\Delta} k_2 \\ \bar{\Delta} k_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta} f_1 \\ \bar{\Delta} f_2 \\ \bar{\Delta} f_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2.87 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_1 \\ \bar{\Delta} N_2 \\ \bar{\Delta} N_3 \end{pmatrix} = \begin{pmatrix} -8.81 \\ -8.81 \\ 59.34 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_1 \\ \bar{\Delta} F_2 \\ \bar{\Delta} F_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta} K_{i1} \\ \bar{\Delta} K_{i2} \\ \bar{\Delta} K_{i3} \end{pmatrix} = \begin{pmatrix} -8.81 \\ -8.81 \\ 63.91 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} F_{h1} \\ \bar{\Delta} F_{h2} \\ \bar{\Delta} F_{h3} \end{pmatrix} = \begin{pmatrix} \bar{\Delta} \bar{K}_1 \\ \bar{\Delta} \bar{K}_2 \\ \bar{\Delta} \bar{K}_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta} M_1 \\ \bar{\Delta} M_2 \\ \bar{\Delta} M_3 \end{pmatrix} = \begin{pmatrix} -8.814 \\ -8.822 \\ 64.031 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} -8.809 \\ -8.817 \\ 63.924 \end{pmatrix}, \qquad (36)$$



(a) $\bar{\Delta} n_1$, $\bar{\Delta} \theta_1$, $\bar{\Delta} c_{h1}$ (b) $\bar{\Delta} n_2$, $\bar{\Delta} \theta_2$, $\bar{\Delta} c_{h2}$ (c) $\bar{\Delta} n_3$, $\bar{\Delta} \theta_3$, $\bar{\Delta} c_{h3}$

(d) $\bar{\Delta} \bar{k}_1$, $\bar{\Delta} c_1$, $\bar{\Delta} \bar{m}_1$ (e) $\bar{\Delta} \bar{k}_2$, $\bar{\Delta} c_2$, $\bar{\Delta} \bar{m}_2$ (f) $\bar{\Delta} \bar{k}_3$, $\bar{\Delta} c_3$, $\bar{\Delta} \bar{m}_3$

(g) $\bar{\Delta} k_{h1}$, $\bar{\Delta} R_1$, $\bar{\Delta} R_{h1}$ (h) $\bar{\Delta} k_{h2}$, $\bar{\Delta} R_2$, $\bar{\Delta} R_{h2}$ (l) $\bar{\Delta} k_{h3}$, $\bar{\Delta} R_3$, $\bar{\Delta} R_{h3}$

Figure 5. The Spatial Effects of Technological Change in the Remote Region

5. Conclusions

This paper proposed a multi-region small-open economic growth model with capital accumulation and endogenous amenity under assumptions of profit maximization, utility maximization, and perfect competition. Different from, for instance, the Solow growth model and the Ramsey model, we used a utility function, which determines saving and consumption with utility optimization without leading to a higher dimensional dynamic system like by the Ramsey approach. We showed how to solve the equilibrium problem. We also simulated the model with the Cobb-Douglas production functions and demonstrated the existence of a unique equilibrium point and examined effects of changes in some parameters. Our comparative statics analysis provides some important insights. For instance, if technological progress occurs in the technologically advanced region, the national output rises; but if technological progress occurs in technologically less advanced region, the national output falls. This hints the possibility that if the central government improves a technologically less advanced region, the national output may fall. Technological progress may either improve or deteriorate the region's trade balance. As we have provided the procedure for simulation and the model involves many functions and parameters, it is straightforward to examine how changes in two or more parameters at the same time can affect the spatial economic structure. As the model is structurally general, it is possible to deal with various national as well as regional issues. Finally, we should mention that it is known that issues related to tax competition among regions has increasingly caused great interests in economic geography (for instance, Andersson and Forslid, 2003, Baldwin and Krugman, 2004, Bayindir-Upmann and Ziad, 2005, Borck and Pflüger, 2006). We can extend the dynamic equilibrium framework proposed in this study to deal with these issues.

Appendix: Proving Lemma 1

The appendix proves Lemma 1. First, we note that k_j, w_j and $f_j(k_j)$ are determined as functions of r^*, which is fixed in the international market. This study is only concerned with equilibrium as a genuine dynamic analysis is too complicated. At equilibrium, by equations (17) we have

$$s_j(\omega_j) = a_j(\omega_j), \quad 0 \le \omega_j \le L_j. \tag{A1}$$

At equilibrium, we have $\bar{m}_j(\omega_j) = \varepsilon_\pi \bar{y}_j(\omega_j)$, where $\varepsilon_\pi \equiv \varepsilon / (\mu + r^*)$. From this equation and $\lambda \bar{y}_j(\omega_j) = s_j(\omega_j) = a_j(\omega_j) = \bar{k}_j(\omega_j) + \bar{m}_j(\omega_j)$, we have

$$\bar{\lambda} \bar{y}_j(\omega_j) = \bar{k}_j(\omega_j). \tag{A2}$$

Multiplying all the equations in (14) by $n_j(\omega_j)$ and then integrating the resulted equations from 0 to L_j with respect to ω_j, we obtain

$$C_j = \xi \bar{Y}_j, \quad M_j = \varepsilon_\pi \bar{Y}_j, \quad K_{hj} = \eta_h \bar{Y}_j, \quad S_j = \lambda \bar{Y}_j,$$

where we use $\pi = \mu$, $R_{hj} c_{hj} = (r^* + \delta_k) k_{hj} / \alpha_h$ in equations (5) and

$$\eta_h \equiv \frac{\alpha_h \eta}{r^* + \delta_k}, \quad M_j \equiv \int_0^{L_j} n_j(\omega_j)\overline{m}_j(\omega_j)d\omega_j, \quad \overline{Y}_j \equiv \int_0^{L_j} n_j(\omega_j)\overline{y}_j(\omega_j)d\omega_j.$$

From the definition of $\overline{y}_j(\omega_j)$ and $\overline{\lambda}\overline{y}_j(\omega_j) = \overline{k}_j(\omega_j)$, we have

$$\overline{k}_j(\omega_j) = r_j + \mu_0 m - r_0 \Gamma_{jc}(\omega_j), \quad 0 \le \omega_j \le L_j. \tag{A3}$$

From $\overline{\lambda}\overline{y}_j = \overline{k}_j(\omega_j)$, we have

$$\overline{Y}_j = \frac{\overline{K}_j}{\overline{\lambda}}. \tag{A4}$$

From equations (14) and (5), we solve

$$R_j(\omega_j) = \frac{\beta_h \eta \overline{y}_j(\omega_j)}{L_{hj}(\omega_j)}, \quad 0 \le \omega_j \le L_j. \tag{A5}$$

From $\alpha_h R_{hj} c_{hj} = (r^* + \delta_k)k_{hj}$ in equations (5), $\overline{\lambda}\overline{y}_j = \overline{k}_j$ and $R_{hj} c_{hj} = \eta \overline{y}_j$, we have

$$k_{hj}(\omega_j) = \frac{\alpha_h \eta \overline{k}_j(\omega_j)}{(r^* + \delta_k)\overline{\lambda}}. \tag{A6}$$

Multiplying all the equations in (A6) by $n_j(\omega_j)$ and then integrating the resulted equations from 0 to L_j with respect to ω_j, we obtain

$$K_{hj} = \frac{\alpha_h \eta \overline{K}_j}{(r^* + \delta_k)\overline{\lambda}}. \tag{A7}$$

Substituting equations (14) into $U_j(\omega_j)$ and then using $U_j(0) = U_j(\omega_j)$, we have

$$\frac{R_{hj}(\omega_j)}{R_{hj}(0)} = \left(\frac{n_j(\omega_j)}{n_j(0)}\right)^{\overline{\mu}/\eta_0} \left(\frac{\overline{k}_j(\omega_j)}{\overline{k}_j(0)}\right)^{1/\rho\eta_0} \left(\frac{T_{hj}(\omega_j)}{T_{hj}(0)}\right)^{\sigma/\eta_0}, \quad j = 1, \cdots, J. \tag{A8}$$

where we also use $\overline{\lambda}\overline{y}_j = \overline{k}_j$. This equation shows how the land rent, residential density and travel time are interrelated in a given region. Substituting $c_{hj} = A_{hj} k_{hj}^{\alpha_h} L_{hj}^{\beta_h}$ into $(r + \delta_k)k_{hj} = \alpha_{hj} c_h R_{hj}$, we have

$$R_{hj}(\omega_j) = \left(\frac{r^* + \delta_k}{\alpha_h A_{hj}}\right) n_j^{\beta_h}(\omega_j) k_{hj}^{\beta_h}(\omega_j), \tag{A9}$$

where we use equations (6). Substitute equation (A9) into equation (A8)

$$n_j(\omega_j) = n_j(0)\left(\frac{\overline{k}_j(\omega_j)}{\overline{k}_j(0)}\right)^{\beta_k} \left(\frac{T_{hj}(\omega_j)}{T_{hj}(0)}\right)^{\beta_0}, \quad j = 1, \cdots, J, \tag{A10}$$

where we use equations (A6) and

$$\beta_k \equiv \frac{1/\rho - \eta_0\beta_h}{\eta_0\beta_h - \bar{\mu}}, \;\; \beta_0 \equiv \frac{\sigma}{\eta_0\beta_h - \bar{\mu}}.$$

Integrating equation (A10) with respect to ω_j from 0 to L_j, we obtain

$$n_j(0) = \phi_j N_j, \quad j = 1, \cdots, J, \tag{A11}$$

in which

$$\phi_j \equiv \frac{\bar{k}_j^{\beta_k}(0) T_{hj}^{\beta_0}(0)}{\int_0^{L_j} \bar{k}_j^{\beta_k}(\omega_j) T_{hj}^{\beta_0}(\omega_j) d\omega_j}.$$

As $\bar{k}_j(\omega_j)$ and $T_{hj}(\omega_j)$ are explicitly defined as a function of location, we see that ϕ_j is independent of location. Substitute equation (A11) into equation (A10)

$$n_j(\omega_j) = \phi_j N_j \left(\frac{\bar{k}_j(\omega_j)}{\bar{k}_j(0)}\right)^{\beta_k} \left(\frac{T_{hj}(\omega_j)}{T_{hj}(0)}\right)^{\beta_0}, \quad j = 1, \cdots, J. \tag{A12}$$

Hence, we determine the residential density distribution as a function of the region's total population, wage rate, and transportation conditions. As the transportation conditions are exogenous, we now decide the population distributions among regions. Substituting equations (14) into the utility function, we have

$$U_j(\omega_j) = \theta_{0j} \xi^{\xi_0} \left(\frac{\varepsilon}{\mu + r^*}\right)^{\varepsilon_0} \eta^{\eta_0} \lambda^{\lambda_0} \bar{y}_j^{1/\rho} \frac{n_j^{\bar{\mu}}(\omega_j) T_{hj}^{\sigma}(\omega_j)}{R_{hj}^{\eta_0}(\omega_j)}. \tag{A13}$$

Substituting equations (A6) into equations (A9), we have

$$R_{hj}(\omega_j) = \frac{(r^* + \delta_k)^{\alpha_h} \eta^{\beta_h} \bar{k}_j^{\beta_h}(\omega_j) n_j^{\beta_h}(\omega_j)}{\alpha_h^{\alpha_h} \bar{\lambda}^{\beta_h} A_{hj}}.$$

Substitute $\bar{y}_j = \bar{k}_j / \bar{\lambda}$ and the above equations into equations (A13)

$$U_j(\omega_j) = \frac{\theta_{0j} A_{hj}^{\eta_0} \xi^{\xi_0} \eta^{\eta_0 - \beta_h \eta_0} \alpha_h^{\alpha_h \eta_0} \bar{\lambda}^{\lambda_0 - 1/\rho + \beta_h \eta_0}}{(r^* + \delta_k)^{\alpha_h \eta_0}} \left(\frac{\varepsilon}{\mu + r^*}\right)^{\varepsilon_0} \bar{k}_j^{1/\rho - \beta_h \eta_0}(\omega_j) n_j^{\bar{\mu} - \beta_h \eta_0}(\omega_j) T_{hj}^{\sigma}(\omega_j). \tag{A14}$$

Inserting $n_j(0) = \phi_j N_j$ into equations (A14) and using $U_j(0) = U_1(0)$, we obtain

$$\frac{N_j}{N_1} = \bar{\phi}_j, \quad j = 2, \cdots, J. \tag{A15}$$

From equations (A15) and (25), we solve the labor distribution as functions of per-capita wealth as follows

$$N_j = \frac{\bar{\phi}_j N}{\sum_{j=1}^{J} \bar{\phi}_j}, \quad j = 1, \cdots, J, \tag{A16}$$

where $\bar{\phi}_1 = 1.$[12] From $\bar{m}_j(\omega_j) = \varepsilon_\pi \bar{y}_j(\omega_j)$, $\bar{\lambda} \bar{y}_j(\omega_j) = \bar{k}_j(\omega_j)$ and (A3), we have

[12] We can explicitly solve the labor distribution in this simple form because we assume that the housing pro-

$$\overline{m}_j(\omega_j) = \frac{\varepsilon_\pi}{\overline{\lambda}}\left[r_j + \mu_0 m - r_0\,\Gamma_{jc}(\omega_j)\right], \ \ 0 \le \omega_j \le L_j . \tag{A17}$$

Substituting (A17) into (9) yields

$$\sum_{j=1}^{J} r_j N_j - r_0 \sum_{j=1}^{J} \int_0^{L_j} \Gamma_{jc}(\omega_j) n_j(\omega_j) d\omega_j = \left(\frac{\overline{\lambda}}{\varepsilon_\pi} - \mu_0\right) Nm. \tag{A18}$$

Substituting (A12) into (A18) yields (26). As $\overline{k}_j(\omega_j)$ and N_j are functions of m, equation (A19) contains a single variable. Hence, we proved lemma 1.

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duction functions have the same value of α_h. It is straightforward to check that if α_h varies in regions, then we can express N_j as a nonlinear function of N_1, i.e., $N_j = g_j(N_j)$. We thus can determine N_1 by $\sum_j g_j = N$ with $g_1 = N_1$. Hence, we can express the labor distribution as functions of m. As shown below, we can still solve the model when α_h varies in regions. For simplicity, we assume the same value of α_h among regions.

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[1] See also McCallum (1983) and Walsh (2003) for general discussions on money.