ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII „ALEXANDRU IOAN CUZA” DIN IAŞI
Tomul LIV Ştiinţe Economice 2007

IDENTIFYING THE NATURE OF THE SEASONAL COMPONENT. APPLICATION FOR ROMANIA'S QUARTERLY EXPORTS BETWEEN 1990-2006

MARIANA GAGEA*

Abstract

The purpose of this paper is to study the identification methods of the nature of the seasonal component of a time series. These methods are represented by the verifying tests of the unit root for the models of seasonal autoregressive processes: the HEGY test, the Franses test, etc. In practice, it has been demonstrated that the seasonal component is both deterministic and stochastic. The HEGY test allows identifying the nature of seasonal variations, but it is difficult to establish a limit between the two parts. The correct arbitration of the test' results of seasonal autoregressive processes, with and without a deterministic component, makes it possible to choose the appropriate methods of seasonal variations elimination. This aspect is highlighted by analysing the time series defined by Romania's quarterly exports between 1990 – 2006.

Key words: seasonal variations, deterministic seasonality, stochastic seasonality, unit root test, HEGY test.

1 Introduction

Seasonal variations are characterised by successive growths and decreases which, theoretically, are rigorously repeating in an identical manner from a p period to another [Jaba, E., 2002, p. 423]. Thus defined, seasonal variations can be eliminated through deseasoning filters (the moving averages method, the regression method with a time variable), or they can be extracted with seasonal dummy variables (the regression method with dummy variables).

Practice has shown that many economic variables, under observation with periodicity inferior to a year, have seasonal variations that change in time, with irregular range. In such situations, seasonal variations have a stochastic nature, and the stochastic processes generating time series are called seasonal unit root processes [Aquirre, A, 2000, p.4]. An exclusively deterministic approach of the seasonal component in a random universe would neglect an important source of instability of the studied phenomenon.

The nature of the seasonal component is established by testing the seasonal unit root. Some frequently used tests are the HEGY test (Hylleberg-Engle-Granger-Yoo), for quarterly seasonality, and the Franses test, for monthly seasonality. Within literature, the development of the identification methods regarding the nature of the seasonal component of time series has been made possible due to authors who, to a great extent, have also given the name to such tests: S. Hylleberg, R.F. Engle, C.W.J. Granger, H.S. Lee, P.H. Franses, E. Ghysels, D.R. Osborn.

The purpose of this paper is a theoretical and practical presentation of the steps taken to verify the unit root in a series of quarterly seasonality by using the HEGY test. The practical example is based on the definite series of Romania's quarterly exports in the period

* Lecturer, Department of Statistics, Faculty of Economics and Business Administration, "Alexandru Ioan Cuza" University, Iasi, e-mail: mariana.gagea@uaic.ro

1990 - 2006. Depending on the seasonality nature, the elimination quality of seasonal variations through known filters is observed.

Due to their novelty, the proof tests of the seasonal, quarterly or monthly unit root are implemented in a reduced number of time series analysis programmes. In this paper, the JmulTi programme is used, which has the HEGY test implemented, with an extended alternative for monthly seasonality. Another programme, much more wide-spread than JmulTi, but limited only to unit root testing for quarterly seasonality, is the STATA programme.

The data used in this paper have been taken from the Monthly Statistical Bulletin, The National Institute of Statistics, Romania, n^o 1 / 1991-2007.

2 Testing the seasonal unit root with the HEGY test

Establishing the nature of seasonal variations by unit root testing is a relatively recent datum. The first applications of these tests were on time series with quarterly seasonality, and they were performed by Hasza-Fuller (1981), then by Dickey-Hasza-Fuller (1984) and developed soon after that by Hylleberg-Engle-Granger-Yoo (1990) [Bourbonnais, R., 2004, p.169]. Nowadays, unit root testing in the case of series with a quarterly seasonality is often done with the HEGY test.

It is thought that a seasonal time series can be represented by one or several of the following three models [Bourbonnais, R., 2004, p.170]:

a) purely deterministic seasonal model: $y_t = \alpha_0 + \beta \cdot t + \sum_{j=1}^{P-1} \alpha_j D_{jt} + \varepsilon_t$,

where:

y_t is the observed value of the phenomenon at a *t* moment;

α_0 - constant of the model, respectively ordered at its origin of the linear trend (if necessary);

β - slope of the linear trend;

α_j - seasonal coefficient corresponding to the seasonal period *j*;

D_{jt} - dummy variable related to the seasonal period *j*;

P – periodicity of the seasonal component, for the quarterly seasonality $P = 4$;

ε_t - "white noise" errors, $\varepsilon_t \rightarrow iid(0, \sigma_\varepsilon^2)$.

The model can be extended to polynomial deterministic tendencies with a degree higher than one.

b) random stationary seasonal model: $\phi(L) y_t = \mu_t + \varepsilon_t$,

where:

$\phi(L)$ is the polynomial of the lag operator whose roots are outside the unit circle of the complex plan;

μ_t may be a constant, a tendency or dummy variables related to seasonality;

c) integrated seasonal model: $\phi(L) Y_t = \mu_t + \varepsilon_t$,

where $\phi(L)$ is the polynomial of the inequality operator which has at least one unit root.

Determining the unit roots of the polynomial $\phi(L)$ with the aid of the HEGY test is based upon the seasonal difference filter, having the form: $(1 - L^P) = \Delta_P(L)$,

where:

L is the lag operator, $L^P y_t = y_{t-P}$;

Δ - difference operator, $\Delta y_t = y_t - y_{t-1}$;

P – periodicity of the seasonal component, for the quarterly seasonality $P = 4$.

The polynomial $(1 - L^4)$ is decomposed into $(1 - L^4) = (1 - L)(1 + L)(1 - iL)(1 + iL)$, with the following roots: an un-seasonal unit root, $r_1 = 1$; three seasonal roots with a unit module, $r_2 = -1$, $r_3 = i$, $r_4 = -i$.

In order to test the hypothesis on the existence of a unit root, "auxiliary regression" is used [5, p. 6]:

$$\phi^*(L) y_{4,t} = \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \mu_t + \varepsilon_t \qquad (1)$$

where:

$y_{k,t}$ are filtered series of different seasonal components of the series y_t, $y_{k,t} = \phi_k(L) y_t$, for $k = 1$ and 4; $y_{k,t} = -\phi_k(L) y_t$, for $k = 2$ and 3;

$\phi_k(L)$ - usual filters by which the seasonal roots of a unit module are eliminated, others that the one corresponding to the component *k*: $\phi_1(L) = 1 + L + L^2 + L^3$; $\phi_2(L) = -(1 - L + L^2 + L^3)$; $\phi_3(L) = -(1 - L^2)$; $\phi_4 = 1 - L^4$;

μ_t may be a constant, a tendency or dummy variables related to seasonality;

ε_t - "white noise" errors, $\varepsilon_t \to iid(0, \sigma_\varepsilon^2)$.

The parameters of equation (1) are estimated through the last squares method.

The order of the polynomial $\phi^*(L)$ is determined either by the gradual growth of the inequalities until "white noise" errors are obtained, or by minimising the values of the model comparison criteria (AIC – Akaike Information Criterion or SIC – Schwartz Information Criterion) [2, p.171].

The test principle. One admits that $\phi(L)$ has a unit root in r_k only if $\pi_k = 0$.

The null hypothesis of the test includes the significance of the auxiliary regression parameters π_k. The significance of the parameters π_1 and π_2 is verified by testing the individual hypotheses [7]:

a) $H_0^a: \pi_1 = 0$; b) $H_0^b: \pi_2 = 0$.

The statistical test. The values of the statistics *t* are calculated and compared with the critical values tabled by Franses and Hobjin (1997), for v degrees of freedom and a threshold of significance α, in general $\alpha = 0{,}05$.

The decision rule. If the calculated test value is higher than the theoretical value ($t_{\pi_k} > t_{th}$), the null hypothesis H_0 is accepted.

If the null hypothesis is admitted $H_0: \pi_1 = 0$, the time series has an un-seasonal unit root, $r_1 = 1$, which is eliminated through the regular difference filter (*1-L*). If one accepts $\pi_2 = 0$, the series has an un-seasonal root $r_1 = -1$, and it is filtered with the operator (*1+L*).

The parameters π_3 and π_4 can be individually checked, but testing the conjugated null hypotheses is preferred, for which the statistical test becomes more powerful:

c) $H_0^c: \pi_3 = \pi_4 = 0$; d) $H_0^d: \pi_2 = \pi_3 = \pi_4 = 0$; e) $H_0^e: \pi_k = 0$, $\forall\ k = \overline{1,4}$.

The statistical test. The conjugated null hypotheses are verified with the F test, corresponding to the Fisher test.

The decision rule. If the test calculated value is lower than the theoretical value ($F_{\pi_k} < F_{th}$), the null hypothesis H_0 is accepted, under the conditions of the accepted risk α.

If the parameters π_3 and π_4 are null, the time series has two seasonal roots of a unit module, $r_3 = i$, $r_4 = -i$, which are eliminated through the filter $(1 + L^2)$.

The asymptotic distributions of the statistics t and F differ according to the deterministic term included in the model (trend, constant, dummy variables). The critical values of the tests t and F have been tabled for each model, taking into account thresholds of significance and samples of different sizes.

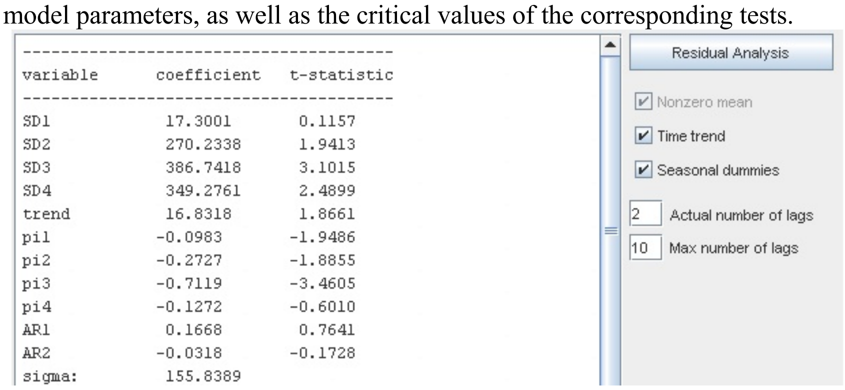
Testing the deterministic term

The trend parameters and the coefficients of the dummy variable which could be included in the model are tested with the aid of the t-Student test, according to the decision hypotheses and rules corresponding to the bilateral test. If one rejects the null hypothesis formulated on the coefficients of the dummy variables, and to the seasonal coefficients respectively, the conclusion is that seasonality has a deterministic component.

3 Identifying the nature of seasonal variations of Romania's quarterly exports

The dynamic form of the series defined by Romania's quarterly exports between 1990-2006 is analysed through graphical and numerical methods. The methods used are the linear chronogram and the Fisher test. Both methods show the presence of the seasonal tendency and component in the time series under analysis.

The purpose is to identify the nature of seasonal variations, with the aid of the HEGY test, by using the JMulti programme. The model with a linear trend and dummy variables is estimated. The output of the HEGY test contains the estimated and tested values of the model parameters, as well as the critical values of the corresponding tests.



variable	coefficient	t-statistic
SD1	17.3001	0.1157
SD2	270.2338	1.9413
SD3	386.7418	3.1015
SD4	349.2761	2.4899
trend	16.8318	1.8661
pi1	-0.0983	-1.9486
pi2	-0.2727	-1.8855
pi3	-0.7119	-3.4605
pi4	-0.1272	-0.6010
AR1	0.1668	0.7641
AR2	-0.0318	-0.1728
sigma:	155.8389	

Source: Processed with the JmulTi programme, according to the Monthly Statistical Bulletin, National Institute of Statistics, nº 1/1991 – 2007

Fig. 1 Estimated and tested values of the parameters of the model with a linear trend and dummy variables

For an admitted risk of 5%, the results of the HEGY test applied to the series of "Romania's quarterly exports" lead to the following decisions:

- the hypothesis on the existence of a deterministic part in the seasonal component is accepted, and the seasonal coefficients 3 and 4 are significantly different from zero (figure 1): $(|t_{S3}| = 3{,}10) > (t_{th} = 1{,}96)$ and $(|t_{S4}| = 2{,}48) > (t_{th} = 1{,}96)$;

- the hypothesis on the existence of the linear trend is rejected (figure 1): $(t_b = 1{,}86) < (t_{th} = 1{,}96)$.

The model with dummy variables and without a linear trend is estimated and tested. According to the results (figure 2), the seasonal component has a deterministic part, represented by the seasonal coefficient S_3 statistically significant: $(|t_{S3}| = 2{,}42) > (t_{th} = 1{,}96)$.

The null hypotheses regarding the roots π_1 and π_2 are accepted; thus, the model has a unit root, the seasonal variations contain a stochastic part and have a semester periodicity (figure 3):

$(t_{\pi_1} = -0{,}54) > (t_{th} = -2{,}84)$, $(t_{\pi_2} = -1{,}86) > (t_{th} = -2{,}83)$,

$(F_{\pi_3\pi_4} = 8{,}36) > (F_{th} = 6{,}57)$, $(F_{\pi_2\pi_3\pi_4} = 7{,}50) > (F_{th} = 5{,}95)$,

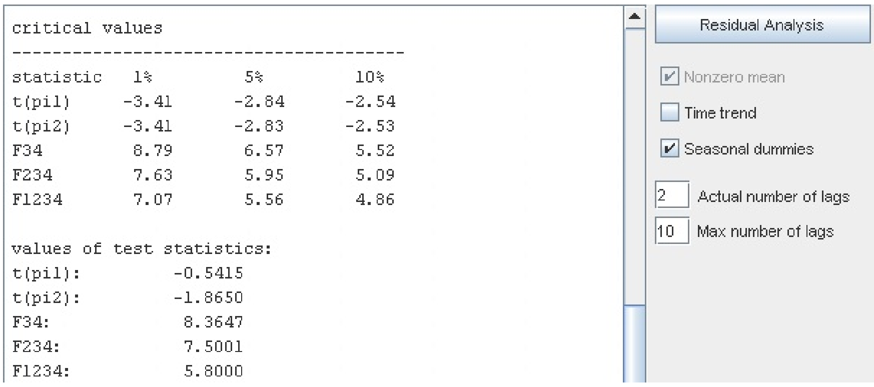
$(F_{\pi_1\pi_2\pi_3\pi_4} = 5{,}80) > (F_{th} = 5{,}56)$.

variable	coefficient	t-statistic
SD1	-149.4957	-1.1951
SD2	153.2561	1.1817
SD3	278.5460	2.4182
SD4	203.2950	1.6733

Source: Processed with the JmulTi programme, according to the Monthly Statistical Bulletin, National Institute of Statistics, n° 1/1991 – 2007

Fig. 2 Estimated and tested values of the seasonal coefficients for the model without a linear trend

According to the results of the HEGY test, the seasonal component has both a deterministic and a stochastic nature. Under these circumstances, seasonal variations are eliminated, in a first stage, through the method of deseasoning moving averages, the additive variant [Jaba, E. 2002, pp. 445-447; Vaté, M., 1993, p. 185]. The HEGY test is applied for the time series which has been corrected of all seasonal variations. The results confirm that the deterministic component of seasonality has thus been eliminated, and the unit roots have been maintained r_1 and r_2: $(t_{\pi_1} = -2{,}297) > (t_{th} = -3{,}39)$, $(t_{\pi_2} = -2{,}165) > (t_{th} = -2{,}82)$. A thorough filtering would assume the elimination of the module unit roots through the filter $(1-L)(1+L) = (1-L^2)$. However, in practice, the quarterly seasonal difference filter $(1-L^4)$ is applied automatically. There is a test of the presence of the seasonal unit root within the deseasoned and filtered series. The goal to eliminate mixed seasonal variations has been achieved.



critical values

statistic	1%	5%	10%
t(pi1)	-3.41	-2.84	-2.54
t(pi2)	-3.41	-2.83	-2.53
F34	8.79	6.57	5.52
F234	7.63	5.95	5.09
F1234	7.07	5.56	4.86

values of test statistics:

t(pi1):	-0.5415
t(pi2):	-1.8650
F34:	8.3647
F234:	7.5001
F1234:	5.8000

Source: Processed with the JmulTi programme, according to the Monthly Statistical Bulletin, National Institute of Statistics, n° 1/1991 – 2007

Fig. 3 Estimated and tested values of the roots π_k for the model with dummy variables and without a linear trend

The variant of the model without dummy variables is analysed, and the result is a seasonality of an exclusively stochastic nature, having all the seasonal roots of a unit module present. The appropriate filter for the elimination of seasonal variations is the seasonal difference operator $(1-L^4)$. The HEGY test is applied to the series thus filtered, and the conclusion is that the seasonal variations have been completely eliminated.

One may notice that the seasonal variations are completely eliminated both through the seasonal coefficients mixed variant – the seasonal difference filter, and through the exclusive variant of the seasonal difference filter.

Choosing one of the variants for the elimination of the seasonal component is a very delicate matter. The variant of the seasonal component of a stochastic nature has offered clearer results, and the elimination of variants was simpler and faster; however, one cannot neglect the deterministic component. The disadvantages of the two approaches are that the elimination of seasonal variants in two stages is very difficult to apply, and the seasonal difference operator $(1-L^4)$ induces variation range higher than the moving averages.

4 Conclusions

Knowing the nature of the seasonal component in a time series is very important in order to choose the appropriate methods to treat the series. Both the elimination of seasonal variations and their modelling or forecasting is done through specific methods, which differ according to the nature of the seasonal component.

Depending on the results of the seasonal root testing and of the seasonal coefficients, the seasonal component may be deterministic, stochastic or mixed. In order to eliminate seasonal variations, the stochastic part applies a seasonal difference operator, according to the existing seasonal roots; the deterministic part is filtered by moving averages or is isolated by dummy variables.

In the example presented, the deterministic component of seasonality seems rather weak; thus, the complete variation filtering can be explained through the difference operator $(1-L^4)$. In practice, there are frequent situations in which the deterministic and stochastic components of seasonality are equally strong and need to be treated simultaneously.

It is necessary to identify the seasonal roots of a unit module in order to establish the form of the filter, as the seasonal difference filter $(1-L^4)$ proposed by Box-Jenkins to eliminate quarterly stochastic seasonality achieves its goal only if all the seasonal roots are present. Otherwise, a super-filtering of the series is performed, having different effects on the results of the analysis.

References

Aquirre, A, *Tests de raices unitarias estacionales en la serie cronológica de los precios del novillo en el mercado de liniers* (Argentina), July 2000, http://www.cedeplar.ufmg.br/pesquisas/td/TD%20140.doc, accessed on June 2, 2006

Bourbonnais, R., Terraza, M., *Analyse des series temporelles. Aplication à l'economie et à la gestion*, DUNOD, Paris, 2004

Hyllebereg, S., *Modelling seasonality,* Oxford University Press, New York, SUA, 1992

Jaba, E. *Statistica* (*Statistics*), 3rd edition, Economic Publishing House, Bucharest, 2002

Pichery, M.-C., Ouerfelli, C., La non stationarité dans les séries saisonniéres, 1998, *http://econpapers.repec.org/paper/latlateco/1998-09.htm*, accessed on March 15, 2005Vaté, M., *Statistique chronologique et prevision*, Economic Publishing House, Paris, 1993