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NONLINEAR MARKET DYNAMICS BETWEEN STOCK RETURNS AND TRADING VOLUME: EMPIRICAL EVIDENCES FROM ASIAN STOCK MARKETS

Wu-Jen Chuang*, Liang-Yuh Ou-Yang**, Wen-Chen Lo***

Abstract

Recent empirical researches report that nonlinear dynamics is present in asset returns because of noise traders involved in the market. This study examines whether there exists any nonlinear dynamics in Asian stock markets. We employ the smooth transition autoregressive model with the percentage change in trading volume as the transition variable to capture the nonlinear movement between stock returns and trading volume in Taiwan, Hong Kong, Singapore, and Korea stock markets. The results show nonlinear dynamics exist between stock returns and trading volume in the stock market. Moreover, trading volume plays an important role for the cyclical movements in the stock market.

Key words: Nonlinear dynamics, Cyclical behavior, Stock market returns, Trading volume, STAR models
JEL Classification: C11, F30, G15

1. Introduction

Stock prices are believed to be sensitive to the relevant economic news. Based upon the capital asset pricing model (CAPM) and the arbitrage pricing theory (APT), stock market returns can be predicted by financial and macroeconomic variables. Investors can reward excess returns by taking systematic risks, but not earn extra premium by bearing diversifiable risk.[1] However, no satisfactory model can argue that there is linear relation between stock returns and macroeconomic factors. A number of increasing empirical evidences challenge the CAPM and APT. First, theoretically investors are thought to be rational under CAPM, but some empirical results show that investors are not rational all the time and that irrational investment behavior has influences on the price formation of securities. For example, De Long et al.(1990) propose a model to show that investors' irrational beliefs have influences on the price formation of assets. They point out that the irrational beliefs would drive the stock prices further away from the fundamental values, and

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the deviations of stock prices become more extreme. Moreover, Lee et al. (2002) employ a GARCH-M model to show that the conditional volatility and excess returns of the stock market are affected by investor sentiment.

Second, both CAPM and APT predict asset returns with priced variables in a linear way. However, some studies have already presented that the business cycle shows nonlinear characteristics and economic variables exhibit asymmetric processes during contraction and expansion in the economy (Keynes, 1936; Teräsvirta and Anderson, 1992; Öcal and Osborn, 2000).[2] Recent studies put more attention on the stock returns of which dynamics are characterized by nonlinear behavior in the business cycle. For example, Leung et al. (2000) predict the international stock returns by artificial neural network method. Some papers examine U.S. stock market by a Markov switching model (Turner et al. 1989; Perez-Quiros and Timmermann, 2000). Sarantis (2001) and McMillan (2003, 2004, 2005,2007) employ the smooth transition autoregressive (STAR) model to examine non-linear behavior in the international stock markets.

Moreover, several researches account for different reasons of nonlinearities in financial markets. The main explanations include that heterogeneous beliefs between informed investors and noise traders (Brock and LeBaron, 1996; Shleifer, 2000), different investment horizons, geographical locations, and risk profiles varied among investors (Peters, 1994), market frictions and transaction costs (Dumas, 1992; Sercu et al. 1995), and different economic growth (Cecchetti et al. 1990). Sarantis (2001) states that smooth transition models are not only suitable to explore the nonlinear and cyclical behavior of stock returns, but also appropriate to explain smooth transition during regime changes due to heterogeneous beliefs, varying learning speeds, and different investment horizons between investors.

Many studies investigate nonlinear behavior of international stock prices by the STAR model (McMillan 2001, 2003, 2004, 2005, 2007; Sarantis, 2001).[3] But so far, utilizing the STAR models to highlight the interactions between stock returns and trading volume is few even though past trading volume providing some valuable information about stock returns has been recognized by financial academics(McMillan 2007). Blume et al. (1994) present traders can learn valuable information of stocks by past prices and past volume, and argue that stock returns and trading volume are jointly determined by the same market dynamics. Datar et al. (1998) uncover the relation that low volume firms earn higher future returns, and high volume firms gain lower future returns. Conrad et al. (1994) find low trading volume stocks exhibit price reversal pattern, but high volume stocks easily show return continuation by studying weekly data. Moreover, Lee and Swaminathan (2000) illustrate the interaction between price momentum, reversal, and trading volume by momentum life cycle hypothesis and suggest investor to make profit by the momentum investing strategy based on past price and volume information. Shiller (2000) also proposes the feedback loop theory to explain the relationships among stock returns, investor sentiment, and trading volume during the stock market cycle. Furthermore, Tetlock (2007) constructs a straightforward measure of media content to proxy for investor sentiment and finds the interactions among media content, market prices and trading volume.

To sum up, both theoretical models and empirical results mentioned above demonstrate that there is interrelationship between stock returns and volume during the stock market cycle. Hence, the objective of this paper is to examine nonlinear dynamics between stock returns and trading volume and to consider trading volume as the transition variable to examine the nonlinear stock market dynamics. The remained of the paper is organized as follows. In the next section, we present the specification and estimations of STAR models.

In section 3, we show and interpret our empirical results. In section 4, we compare the out-of-sample forecasting performance of the nonlinear model with that of the linear model. In the final section, we conclude our findings.

2. Specification and Estimation of STAR Models

The paper is to exploit the potential nonlinear and cyclical behavior between stock returns and trading volume. We employ the STAR models which allow the transition variable to cause a slow change between different regimes to investigate nonlinear relation in stock market (Teräsvirta and Anderson, 1992; Teräsvirta, 1994). Moreover, we consider trading volume as the transition variable to examine the nonlinear dynamics between stocks returns and trading volume.

2.1. Trading volume as the transition variable

There are some explanations about the role of trading volume in the market. Prior researches interprate trading volume as a liquidity proxy. Based upon the liquidity hypothesis, relatively low volume stocks are less liquid, but gain higher expected return (Amihud and Mendelson, 1986; Datar et al., 1998; Brennan et al., 1998). Some researches find that trading volume contains valuable information of stocks and propose change in volume can measure abnormal activity. Lee and Swaminathan (2000) state that low (high) volume stock display characteristics of value (glamour) stocks. The reason why low (high) volume stocks usually gain higher (lower) future returns is that investors mispercept about future earnings of those stocks.

Besides, Lee and Swaminathan (2000) propose the momentum life cycle hypothesis to show that trading volume and stock returns have interrelation in the stock market cycle. Shiller (2000) illustrates how investor expectations for future market performance, related information of stocks, and trading volume are driven by the mechanism of the feedback loop or the self-fulfilling prophecy.

Hence, the trading volume is an important factor to trigger off the operations of the market cycle. When we try to find out nonlinear dynamic in the stock market, the significant influence of trading volume on stock returns should be considered. That is, in this study we employ the trading volume as the transition variable to examine the nonlinear dynamics of stock markets.

2.2. Specification of models

The STAR family of models has two particularly useful forms. One is the logistic STAR (LSTAR) model which describes a situation where different states of an economy have different dynamics and the transition from one to the other is smooth. The other is the exponential STAR (ESTAR) can explain the similar dynamic structure of different phases of an economy, but the middle ground can have different dynamics (Teräsvirta and Anderson, 1992)

The advantage of employing STAR model in studying financial market is to take considerations that individuals can be impossible to react simultaneously to certain information. For example, when economic indicators show bad signals, investors in the market would not take the same decisions at the same time. Some sell their investments immediately, but some keep them for a while. Heterogeneities among investors may result from the different risk aversion, experiences in processing information, investing objective, and time to get valuable information. The STAR model allows there are continuous states, not abrupt structural change, between the extremes. Hence, STAR models are more suitable and realistic to process market dynamics. Moreover, STAR models can explain different states during the market cycle, such as, the bull and bear markets. Investors could have different strategies in the bull and bear market, so the market dynamic could be different in the bull and bear market. The LSTAR model is good to describe such characteristics. Furthermore, extremes, such as peak and trough of stock markets, have similar dynamic, but the mid-ground, periods of price going up continuously and of price going downwards gradually, show different dynamics. During the peak and trough of stock markets, returns show characteristics of reversal; otherwise, returns exhibit price momentum behavior. The ESTAR model is a suitable model to account for such pattern.

The STAR model is defined as

$$r_t = \alpha_0 + \alpha_1^{'} x_t + (\beta_0 + \beta_1^{'} x_t) F(S_{t-d}) + \varepsilon_t \tag{2.1}$$

where r_t is the market stock return, $x_t = (r_{t-1}, r_{t-2}, ..., r_{t-k})^{'}$, $\alpha_1 = (\alpha_1, \alpha_2, ..., \alpha_k)^{'}$, $\beta_1 = (\beta_1, \beta_2, ..., \beta_k)^{'}$, $\varepsilon_t \overset{iid}{\sim} N(0, \sigma^2)$, F is the transition function, S_{t-d} is the transition variable, and d is the delay parameter.

The transition function can be a logistic form:

$$F(S_{t-d}) = \{1 + \exp[-\gamma(S_{t-d} - c)]\}^{-1},\ \gamma > 0 \tag{2.2}$$

where γ is the smooth parameter, measuring the transition speed from one regime to the other; c indicates the threshold. When γ is great than 0, the degree of autoregressive decay depends on the transition variable, S_{t-d}. When S_{t-d} is far above the threshold, the value of transition function would approach 1; and when S_{t-d} is far below the threshold, the value of transition function approaches 0.[4] Hence, LSTAR model characterizes asymmetric processes of market cycles.

The transition function can also be an exponential form:

$$F(S_{t-d}) = 1 - \exp[-\gamma(S_{t-d} - c)^2],\ \gamma > 0 \tag{2.3}$$

When γ is between 0 and ∞, the degree of autoregressive decay depends on the transition variable, S_{t-d}. As S_{t-d} is close to the threshold, the value of transition function approaches to 0. As S_{t-d} moves farther away from the threshold, the value of transition function is 1. The ESTAR model suggests similar dynamic for low and for high values of the transition variable, but different dynamic for mid-range of the transition variable.

We follow the modeling procedure as supported by to Teräsvirta and Anderson (1992), Teräsvirta (1994), and Sarantis (2001) to build our STAR models.

Step 1: Specify a linear AR model. We choose the lag length, *p*, of AR model to determine the maximum value of k by Ljung-Box Q statistic for autocorrelation.

Step 2: Test the linearity of STAR models for different value of delay parameter, *d*, of the transition variable. We estimate the following auxiliary equation.

$$r_t = \alpha_0 + \alpha_1' x_t + \sum_{j=1}^{p} \alpha_{2j} x_{t-j} s_{t-d} + \sum_{j=1}^{p} \alpha_{3j} x_{t-j} s_{t-d}^2 + \sum_{j=1}^{p} \alpha_{4j} x_{t-j} s_{t-d}^3 + u_t \qquad (2.4)$$

The null hypothesis for linearity test is $\alpha_{2j} = \alpha_{3j} = \alpha_{4j} = 0$, j=1,…, p . We chose d when the linearity is rejected with the smallest test statistic of p-value.

Step 3: Choose the appropriate model of STAR family by testing following restrictions:

$$H_{04} : \alpha_{4j} = 0, \; j = 1,..., p. \qquad (2.5)$$

$$H_{03} : \alpha_{3j} = 0 / \alpha_{4j} = 0, \; j = 1,..., p. \qquad (2.6)$$

$$H_{02} : \alpha_{2j} = 0 / \alpha_{3j} = \alpha_{4j} = 0, \; j = 1,..., p \qquad (2.7)$$

If (2.5) is rejected we choose the LSTAR model. If (2.5) is not rejected and (2.6) is rejected, we choose the ESTAR model. If (2.5) and (2.6) are accepted and (2.7) is rejected, we select the LSTAR model.

3. Empirical Results

3.1. The data

We investigate weekly stock index returns and trading volume of four Asian countries to explore their nonlinear dynamic relationship[5]. There are Taiwan Weighted Stock Index, Hong Kong Hang Seng Index, Singapore Straits Times Index, and Korea Composite Index[6]. The data for Hong Kong, Singapore, and Korea are from the website Yahoo! Finance[7]. The data for Taiwan Stock Index is from the Taiwan Economic Journal (TEJ) data bank. All the index returns are presented in natural logarithms. To avoid the problem of different scales in stock returns and trading volume, we employ the percentage change of trading volume as the transition variable to uncover the nonlinear dynamic relationship between stock market returns and trading volume.

3.2. Unit root tests and descriptive statistics

The unit root tests of stock index returns and the percentage change in trading volume are reported in Table 1. The unit root tests for those series show the evidences of stationarity. Table 2 shows some descriptive statistics for returns and for percentage change in trading volume. We report their means, standard deviations, maximum, and minimum value. Because we will take percentage change in trading volume to be the transition variable of the STAR models, we need to find the value of the threshold of percentage change in trading volume, and make sure the value of the threshold between the maximum and minimum value. We can find percentage change in trading volume of four countries vary quite small.

Table 1. Unit root tests for stock returns and for the percentage change in trading volume

Unit root tests for stock returns					**Unit root tests for the change percentage in trading volume**			
Country Statistic	Taiwan	Hong Kong	Singapore	Korea	Taiwan	Hong Kong	Singapore	Korea
Wtd.Sym.	-10.023***	-9.403***	-9.260***	-7.704***	-11.639***	-7.516***	-9.793***	-7.994***
Dickey-F	-9.972***	-9.356***	-9.205***	-8.277***	-11.639***	-7.521***	-9.753***	-8.215***
PP	-2011.2***	-280.8***	-601.4***	-511.7***	-1870.8***	-274.4***	-583.8***	-407.9***

*** represents 1% level of significance. Wtd.Sym, Dickey-F, and PP are the Weighted Symmetric test, Dickey-Fuller test, and Phillips-Perron test, respectively.

3.3. Tests for linearity and selection of STAR models

The results for the maximum lag of the AR models and for the linearity tests are presented in Table 3. We find out the maximum lag of the AR models by Ljung-Box Q statistic (LB) for autocorrelation. Most countries we estimating have relatively long lag length.

Table 2. Descriptive statistics for stock returns and for the percentage change in trading volume

Descriptive statistics for stock returns					**Descriptive statistics for the percentage change in trading volume**			
Country Statistic	Taiwan	Hong Kong	Singapore	Korea	Taiwan	Hong Kong	Singapore	Korea
Mean	0.002	0.001	0.0004	0.003	0.004	0.005	0.011	0.004
Std.Dev.	0.041	0.025	0.031	0.044	0.372	0.273	0.505	0.237
Max	0.220	0.108	0.199	0.174	3.587	1.131	2.655	-1.433
Min	-0.253	- 0.078	-0.255	-0.149	-5.010	-1.290	-2.229	1.464

When the linearity test is rejected at 5% level of significant, those countries suggest nonlinear dynamics in stock markets. Results suggest the delay parameters of transition variable in most countries are 0, except for Hong Kong. The evidences imply that the percentage change in trading volume at the same period can induce nonlinear dynamics of stock returns in all of countries we investigating but Hong Kong. We can argue that there is interrelationship between stock returns and the percentage change in trading volume in the current period in those countries. Only the evidence of Hong Kong shows that two-period lagged percentage change in trading volume can result in nonlinear dynamics of stock returns.

Table 4 reports the results of model specification. Based upon the selection criteria mentioned in section 2, nonlinear models of all countries are determined. We will employ the LSTAR model in Taiwan and Korea to explore their asymmetric dynamics of the stock markets. Moreover, we explain nonlinear dynamics of the stock market of Hong Kong and Singapore by the ESTAR model.

Table 3. Tests for linearity

Country	Maximum k[a] LB(υ)[b]	Minimum P-value over $0 \le d \le 8$[c]	Delay parameter
Taiwan	k=32 LB(32)= 18.734 P-value= 0.662	0.000	d=0
Hong Kong	k=12 LB(38)= 6.743 P-value= 0.663	0.024	d=2
Singapore	k=40 LB(38)= 27.043 P-value= 0.829	0.000	d=0
Korea	k=40 LB(40)= 24.087 P-value= 0.935	0 .003	d=0

Notes:
a: First, we estimate linear AR models of different orders. The maximum value of k is selected by Ljung-Box Q statistic for autocorrelation.
b: LB(υ) is the Ljung-Box statistic for υ order autocorrelation in the AR model.
c: Choose d with the lowest P-value over the range 0 to 8.

Table 4. Specification of the nonlinear model[a]

Country	H_{04} **b**	H_{03} **b**	H_{02} **b**	**Type of model**
Taiwan	0.0025	0.0091	0.0000*	LSTAR
Hong Kong	0.5434	0.0337*	0.0401	ESTAR
Singapore	0.0371	0.0002*	0.1163	ESTAR
Korea	0.1683	0.0346	0.0150*	LSTAR

Notes:
a: The value listed on the column of H_{04}, H_{03} and H_{02} are p-value for testing model specifications.
b: Equation (6), (7) and (8) in the section 2.

3.4. Estimates of the nonlinear models

The estimates of STAR models are processed by the method of nonlinear least squares and reported in Table 5. Moreover, we follow the suggestions of academics to estimate γ by dividing the standard deviation of r_t, $\sigma(r)$, for the LSTAR model, and by dividing the variance of r_t, $\sigma^2(r)$, for the ESTAR model (Granger and Teräsvirta, 1993; Teräsvirta, 1994). Hence, γ is scale-free and easier to interpret.

From the Table 5, estimates of the parameters of transition function, β, are significant and show strong evidences of nonlinear models. Most important, the smoothing parameters, γ, of all models are highly significant and γ is quite small in most countries. The results suggest there is slow transition between regimes in those countries. However, comparing with other countries, Singapore has rather quick transition speed between regimes. Besides, the thresholds of all models are highly significant at the 5% level.

We plot the estimated transition functions of STAR models for stock returns in Taiwan, Hong Kong, Singapore, and Korea. From Figure 1(a) to Figure 4(b), we show the shapes of the transition functions of four countries. Each points can indicate what value of transition function has obtained and how frequently. For example, from Figure 1(a) and 1(b), we can see readily the values of transition function in Taiwan are within the two thresholds most time. Figure 2(a), 2(b), 3(a), and 3(b) also show the transition functions are normally to be one in Hong Kong and Singapore. Besides, from 4(a) and 4(b), we find values of the transition functions are usually to be zero in Korea.

3.5. Dynamic behavior

It is different to interpret the estimated coefficients of STAR model from Table 5. However, we can get more information about dynamic behavior of estimated models by examining the characteristic roots of polynomial models. We compute the roots of the STAR models by solving the following characteristic polynomial (Teräsvirta and Anderson,1992) for F=0, 1.

$$\lambda^k - \sum_{j=1}^{k} (\alpha_{1j} + \beta_{1j} F)\lambda^{k-j} = 0 \qquad (3.8)$$

When F=0, which means the lower or contraction regime in the LSTAR model and the middle range in the ESTAR model. When F=1, which explains the upper or expansion regime in the LSTAR model and the outer (contraction or expansion) regime in the ESTAR model. The Table 6 shows the most prominent roots of estimated models for each regime in four countries.

Table 5. Estimates of the nonlinear models[a]

Taiwan: LSTAR

$$r_t = -0.025 + 0.055 r_{r-1} + 0.223 r_{r-2} + 0.074 r_{r-3} + 0.164 r_{r-11} - 0.176 r_{t-32} + (0.061 - 0.169 r_{r-2}$$

(-7.782) (2.500) (5.818) (3.424) (4.116) (-3.922) (9.979) (-2.139)

$$-0.184 r_{t-11} - 0.141 r_{r-14} + 0.304 r_{r-32}) \times \langle 1 + \exp\{-1.922(1/\sigma(r)(s_t - 0.067)\ \}\rangle^{-1}$$

(-2.447) (-3.519) (3.475) (5.967) (1.812)

VAR / VAR_l =0.816

Hong Kong: ESTAR

$$r_t = 0.420 + 17.812 r_{r-6} - 14.287 r_{r-8} - 7.420 r_{r-12} + (-0.418 - 17.657 r_{t-6} + 14.191 r_{r-8} + 7.304 r_{r-12})$$

(7.072) (5.982) (-9.121) (-4.121) (- 7.036) (-5.926) (8.969) (4.067)

$$\times \langle 1 - \exp\{-13.002(1/\sigma^2(r))(s_{t-2} - (0.637)^2)\ \}\rangle$$

(4.622) (44.950)

VAR / VAR_l =0.844

Singapore: ESTAR

$$r_t = -4.876 - 94.053 r_{r-1} + 204.249 r_{r-3} - 116.012 r_{r-22} - 76.480 r_{r-33} - 41.466 r_{r-40} + (4.877 + 94.132 r_{r-1}$$

(-2.074) (-2.181) (2.338) (-2.082) (- 2.588) (-3.352) (2.074) (2.183)

$$-204.125 r_{r-3} + 116.080 r_{r-22} + 76.572 r_{r-33} + 41.326 r_{r-40})$$

(-2.337) (2.083) (2.591) (2.344)

$$\times \langle 1 - \exp\{-362.823(1/\sigma^2(r))(s_t - (1.196))^2\ \}\rangle$$

(2.446) (168.484)

VAR/VAR_l =0.845

Korea: LSTAR

$$r_t = 0.047 - 0.125 r_{r-1} + 0.094 r_{t-3} + 0.318 r_{r-13} + (-0.065 - 0.334 r_{r-13})$$

(4.335) (-2.765) (2.087) (2.234) (-5.467) (-1.857)

$$\times \langle 1 + \exp\{-4.278(1/\sigma(r))(s_t - 0.552\ \}\rangle^{-1}$$

(2.068) (6.056)

VAR/VAR_l =0.865

Notes:

a: Figures in parenthesis are t-statistics of coefficients. VAR/VAR_l is the ratio of variances for nonlinear and linear models.

For four countries both regimes contain complex pairs of explosive roots. This implies that the stock markets in all countries show cyclical movements during regimes. In Hong Kong and Singapore, the middle regime is dominated by an explosive root, which implies the stock market passing through the thresholds very quick on the way up or down. However, the outer regime is stable, which means the stock market tends to stay there. These arguments also can consist with the Table 5, Figure 2(a), 2(b), 3(a) and 3(b). The transition parameters of Hong Kong and Singapore are large. Moreover, most values of the transition function are near one for these two countries.

For the LSTAR model, Taiwan and Korea show that both upper and lower regimes are stable. The results imply the stock markets of Taiwan and Korea have a tendency to remain at both contraction and expansion phases.

Table 6. The most prominent characteristic roots in regimes

Country	**Regimes[a]**	**Most prominent roots**	**Modulus**
Taiwan	L	0.9620±0.0799i	0.9653
		-0.9404±0.1000i	0.9457
	U	-0.9270	0.9270
		0.9367±0.1885i	0.9555
Hong Kong	M	-1 .6557	1.6557
		1.5402	1.5402
	O	0.8007±0.1499i	0.8146
		0.8303±0.2131i	0.8572
Singapore	M	-1. 4857	1.4857
		1.4620	1.4620
	O	0.9449±0.0590i	0.9467
		- 0.9517±0.0765i	0.9548
Korea	L	0.9147	0.9147
		- 0.8910±0.2283i	0.9198
	U	-0.7245	0.7245
		- 0.6466±0.3506i	0.7355

Note: a. For the ESTAR model, M is the middle range and O is the outer regime; for the LSTER model, L is the lower regime and U is the upper regime.

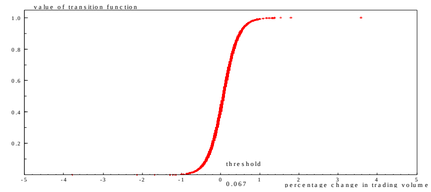

Figure 1(a) The estimated transition function of the LSTAR model for stock returns against the percentage change in trading volume in Taiwan

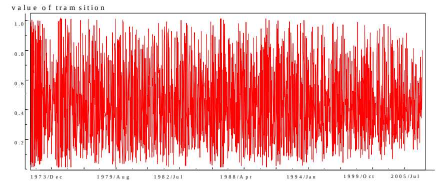

Figure 1(b) The estimated transition function of the LSTAR model for stock returns in Taiwan by trading volume

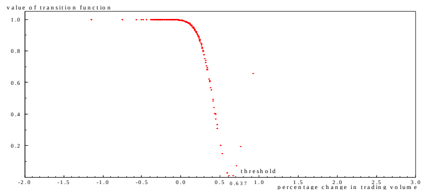

Figure 2(a) The estimated transition function of the ESTAR model for stock returns against the percentage change in trading volume in Hong Kong

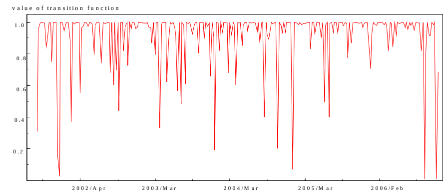

Figure 2(b) The estimated transition function of the ESTAR model for stock returns in Hong Kong

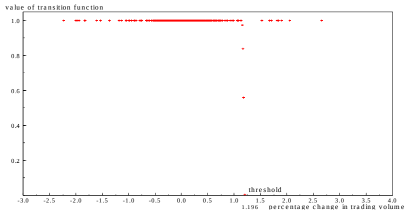

Figure 3(a) The estimated transition function of the ESTAR model for stock returns against the percentage change in trading volume in Singapore

Figure 3(b) The estimated transition function of the ESTAR model for stock returns in Singapore

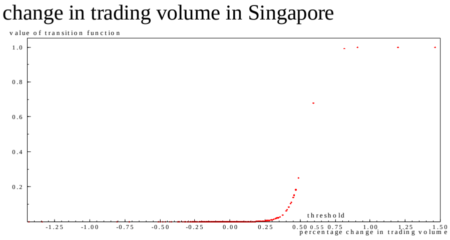

Figure 4(a) The estimated transition function of the LSTAR model for stock returns against the percentage change in trading volume in Korea

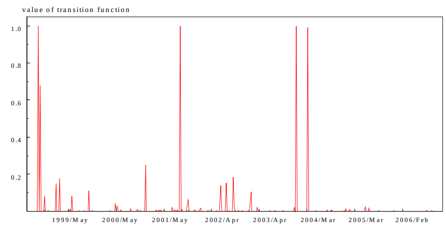

Figure 4(b) The estimated transition function of the LSTAR model for stock returns in Korea

4. Out-of-Sample Forecasting

To evaluate the forecasting abilities of the STAR models, we examine the out-of sample forecasting performance by comparing nonlinear and linear models. First, we re-estimate the STAR models by reducing 12-period data for each country[8]. Then we make out-of-sample forecasting for 1 to 12 periods ahead by re-estimated parameters for STAR models and linear autoregressive (AR) models.

We adopt two criteria to evaluate the accuracy of forecasts. One is the root mean squared error (RMSE). The other is the mean absolute error (MAE). Their definitions are as followings.

Table 7 Out-of-sample forecasts

	Taiwan (trading shares)			Hong Kong			Singapore			Korea		
	Actual	STAR	AR	Actual	STAR	AR	Actual	STAR	AR	Actual	STAR	AR
1-period ahead	-0.029	-0.020	-0.032	0.0053	14.9205	15.0082	0.0047	0.0005	-6.3219	0.0046	0.0504	0.0528
2-period ahead	0.019	0.007	-0.012	0.0208	6.4061	6.5954	-0.0006	0.0043	-4.1066	-0.0094	0.0327	0.0377
3-period ahead	0.006	-0.003	-0.034	-0.0033	0.0976	0.0896	0.0190	0.0006	-2.3336	0.0169	0.0581	0.0645
4-period ahead	-0.002	0.004	-0.018	0.0203	5.7938	5.9447	0.0308	0.0018	-5.2932	-0.0143	0.0368	0.0383
5-period ahead	0.030	0.019	-0.024	0.0048	10.1374	10.4234	0.0065	-0.0014	-6.9889	-0.0025	0.0352	0.0412
6-period ahead	0.000	-0.003	-0.032	0.0069	-4.0289	-4.1189	0.0074	0.0018	1.8430	0.0115	0.0494	0.0518
7-period ahead	0.016	0.012	-0.015	0.0101	19.3404	19.8969	0.0161	0.0041	2.3755	0.0035	0.0374	0.0495
8-period ahead	0.011	0.013	-0.021	0.0244	5.4828	5.6348	-0.0028	-0.0001	-2.9789	0.0107	0.0425	0.0468
9-period ahead	-0.005	0.000	-0.020	0.0075	12.6529	12.7436	0.0084	-0.0027	0.3070	0.0085	0.0368	0.0421
10-period ahead	0.007	0.001	-0.020	0.0153	20.3764	20.9264	0.0244	0.0055	-4.0159	0.0117	0.0530	0.0550
11-period ahead	0.011	-0.001	-0.018	0.004	-0.1565	-0.1524	0.0006	0.0009	-5.2411	0.0067	0.0426	0.0447
12-period ahead	0.002	0.026	-0.035	-0.0029	16.5145	16.9677	0.0092	0.0022	-5.4856	0.0024	0.0509	0.0525
RMSEs		0.0103			11.7839			0.0129			0.0401	
RMSEa			0.0314			12.0535			4.3929			0.0442
MAEs		0.0086			9.6514			0.0102			0.0396	
MAEa			0.0289		14.9205	9.8675			3.9459			0.0439

Note: Actual is the real return. STAR is the estimate from STAR model. AR is the estimate from AR model.
RMSEs is the root mean squared error for STAR models. RMSEa is the root mean squared error for AR models.
MAEs is the mean absolute error for STAR models. MAEa is the mean absolute error for AR models.

$$RMSE = \sqrt{\frac{1}{K}\sum_{i=1}^{K}(r_{t+i}^{f} - r_{t+i}^{a})^{2}} \tag{4.10}$$

$$MAE = \frac{1}{K}\sum_{i=1}^{K}\left|r_{t+i}^{f} - r_{t+1}^{a}\right| \tag{4.11}$$

r_{t+i}^{f} and r_{t+i}^{a} are actual and forecast values respectively at time $t+i$. The forecasting period is from $t+1$ to $t+K$. K is 12, which is the out-of-sample forecasting period. Table 7 summaries the results of out-of-sample forecasting for all countries. We find out values of RMSE and MAE of STAR models are smaller than those of AR models. All results support STAR models have better forecasting performances than AR models.

5. Conclusion

This study examines whether there exists any nonlinear market dynamics between stock returns and trading volume. We examine Taiwan, Hong Kong, Singapore, and Korea stock markets by employing the STAR models. The results are summarized as followings. First, four countries show nonlinear evidences in the stock market cycle. Besides, For Taiwan and Korea, nonlinear stock market dynamics are characterized by the LSTAR models. Stock returns of Hong Kong and Singapore can be described by the ESTAR models.

Secondly, from the characteristic polynomial of most estimated models displays at least one explosive root, which means that stock market cycles in most countries are asymmetric. Moreover, from the out-of-sample forecasting performance of nonlinear models, we also conclude nonlinear models have better forecasting abilities.

Third, we have an important finding, which is trading volume as a trigger to cause nonlinear dynamic cycle of stock returns. In stock markets investors believe "Trading volume signals prices." When trading volume goes to unusual high level, investors would expect the highest prices to happen. Trading volume is a wide-used and informative market statistic. When trading volume shrinks to unusual low level, investors predict the lowest prices to take place. Moreover, valuable information provided by trading volume is useful not only for the individual stock but also for the market index. From the Table 5, the results show all thresholds are highly significant. Strong evidences support that trading volume is good to be the transition variable inducing cyclical dynamics of stock market.

In conclusion, our study shows that investigated countries have nonlinearities and cyclical behavior in stock market returns. Moreover, trading volume plays an important role in the cyclical movements of the stock market. Trading volume is a common market indicator for investors in the stock markets. Investors usually observe the change in trading volume first and then make their investment decisions. Nevertheless, questions remain as to make investment strategies based upon the change in trading volume. We hope to address such topic in our ongoing research.

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Notes:

1. The CAPM (Sharpe,1964 ;Linter ,1965) states that the only systematic risk can be priced and that the expected return of a asset is equal to risk-free rate plus a risk premium multiplied by the asset's systematic risk. Ross (1976) develops the APT which implies that there are multiple risk factors that need to be taken into account when calculating asset returns.
2. Keynes(1936) argues GNP has asymmetric processes during the business cycle. Teräsvirta and Anderson (1992) find nonlinear behavior of the industrial production growth for OECD countries. Öcal and Osborn (2000) investigate nonlinearity of UK consumption and Industrial production.
3. McMillan (2001,2003,2004,2005,2007) employ STAR models to examine non-linear relationship of priced factors and index returns, such as: UK, CAC, Nikki, DAX, Hang Seng, Kuala Lumpur and Singapore Straits; Sarantis, (2001) investigates there are potential linearities in the stock prices of seven major industrial countries.
4. Sarantis (2001) explains that the lower regime, F=0, is the contraction regime and the upper regime, F=1, is the expansion regime in the LSTAR model. McMillan (2003) explains two regimes under the LSTAR model as large returns and small returns.
5. Some economists consider short-frequent economic time series data as too noisy to reflect the cyclical movements of variables (Teräsvirta and Anderson, 1992; Birchenhall et al., 1999). However, if we process yearly or quarterly stock returns and trading volume to find out their nonlinear relationship. The results can not provide valuable information for investors to make decisions. For example, if investors buy stocks based upon last year or last quarter data of trading volume, it is not up-to-date information for most investors. Hence, recent studies concerning about nonlinear dynamics of stock returns employ daily stock index (McMillan,2005). In our study, we exploit valuable information for most investors by weekly data.
6. The sample period for Taiwan Weighted Stock Index is from 1971/1/9 to 2006/11/10, Hong Kong Hang Seng Index is from 2001/7/9 to 2006/11/27, Singapore Straits Times Index is from 1995/1/3 to 2006/11/27, and Korea Composite Index is from 1998/4/27 to 2006/11/27.
7. http://finance.yahoo.com/intlindices?e=asia
8. The re-estimated sample period for Taiwan Weighted Stock Index is from 1971/1/9 to 2006/8/18, Hong Kong Hang Seng Index is from 2001/7/9 to 2006/9/4, Singapore Straits Times Index is from 1995/1/3 to 2006/9/4, and Korea Composite Index is from 1998/4/27 to 2006/9/4.