GLOBAL ECONOMIC GROWTH, ELASTIC LABOR SUPPLY, KNOWLEDGE UTILIZATION AND CREATION WITH LEARNING-BY-DOING

Wei-Bin ZHANG
Ritsumeikan Asia Pacific University
Oita-ken, Japan
wbz1@apu.ac.jp

Abstract

This paper studies issues related to global economic growth, trade patterns and elastic labor supply with capital accumulation and knowledge creation. Trade patterns among countries are determined by free competition and knowledge accumulation is through learning by doing. This study considers knowledge as public good. Knowledge is accessible to all the people in the world, even though different countries apply knowledge with different efficiencies. The countries differ in preference (such as propensities to save and to use leisure time), knowledge utilization efficiency and creativity. First, we show that the dynamics of the J *-country world economy is described by* $(J+1)$ *differential equations. Then, we simulate the motion of the global economy with three economies, respectively called developed, industrializing, and underdeveloped economies. We carry out comparative dynamic analysis with regard to changes in knowledge utilization efficiency.*

Keywords: Global Economic Growth; Trade Pattern; Elastic Labor Supply, Knowledge Utilization and Creation; Learning-by-Doing
JEL classification: F11

1. INTRODUCTION

One of important questions in economics is which country and who will benefit from unprecedented globalization and fast increase in the production capacity. For understanding modern politics and economic life it is essential to understand possible effects of trade upon national and personal income distribution in a globalizing world economy. Economic globalization and income inequality have been widely discussed in both academic and policy forums. Poor countries hope to catch up with rich countries in terms of living standards; rich economies are concerned with their ability to sustain living standards as some developing economies are experiencing fast economic growth. To understand economic growth, sustainability and distribution, it is necessary to know the key determinants of these phenomena. It has been often argued that productivity differences explain much of the variation in incomes across countries, and technology plays a key role in determining productivity [see, Nakajima, 2003; Krugman and Venables, 1995; Manasse and Turrini, 2001; and Agénor, 2004]. It is expected that worldwide main technical changes are determined largely by a few rich countries. Basing on the dataset on imports of technology and total factor productivity (TFP) over 135 years for the OECD countries, Madsen [2007] empirically shows that knowledge spillovers have been a significant contributor to the TFP convergence among the OECD countries over the period of 1870 to 2004. Using a multi-sector version of the Ricardo-Viner model of international trade to empirically identify the effects of technological change and international trade on the US wage premium, Blum [2008] finds that capital was reallocated to sectors where it is relatively complementary to skilled workers. Both capital accumulation and technological change are important for explaining national as well as global economic growth and distribution. The purpose of this study is to build a global economic growth model with endogenous knowledge and capital accumulation.

As globalization is deepening, it is important to provide analytical frameworks for analyzing global economic interactions not only among developed economies, but treating the world as an integrated whole. For instance, it is important to examine how a developing economy like India or China may affect different economies as its technology is improved, wealth is accumulated, population is increased; or how trade patterns may be affected as technologies are further improved or propensities to save are reduced in developed economies like the US or Japan. All these questions are currently well discussed in academics as well as public. As mentioned by Findlay (1984), one topic that was almost entirely absent from the pure theory of international trade was any consideration of the connection between economic growth and international trade in the classical literature of economic theory. Almost all the trade models developed before the 1960s are static in the sense that the supplies of factors of production are given and do not vary over time; the classical Ricardian theory of comparative advantage and the Heckscher-Ohlin theory are static since labor and capital stocks (or land) are assumed to be given and constant over time. It has only been in the last three or four decades that trade theory has made some systematic treatment of capital accumulation or technological changes in the context of international economics. According to the neoclassical growth model a poor economy may catch up with rich ones because of decreasing marginal product of capital.

Most of trade models with endogenous capital and/or knowledge are either limited to two-country or small open economies [see, Wong, 1995; Jensen and Wong, 1998; Obstfeld and Rogoff, 1998; and Zhang, 2008]. We will build a multi-country trade model with capital accumulation and knowledge creation in a perfectly competitive global economy. As far as capital is concerned, our model is influenced by neoclassical trade theory. Trade models with capital movements are originated by MacDougall [1960] and Kemp [1961]. Nevertheless, these early models were limited to static and one-commodity frameworks. The first dynamic growth model of trade with capital accumulation and microeconomic foundation was built by Oniki and Uzawa [1965]. The model takes account of accumulating capital stocks and of growing population within the Heckscher-Ohlin type of model. It was developed in terms of the two-country, two-good, two-factor model of trade. The model is also extended to study the dynamics of capital accumulation and the various balance of payments accounts. There is another alternative specification of trade structure in the growth framework which allows for the existence of international financial markets and for free trade in consumption goods and securities, but not in investment goods [see, Wong, 1995; Jones and Kenen, 1984; Ethier and Svesson, 1986; and Bhagwati, 1991]. It should be noted that Sorger [2002] proposed a trade model based on the Solow approach. Vellutini [2003] proposes a trade model to examine possible poverty traps due to capital mobility. Many two-country, dynamic-optimization models with capital accumulation have been proposed to examine the impact of saving, technology, and various policies upon trade patterns [see, Frenkel and Razin, 1987; Jensen, 1994; Valdés, 1995; and Nishimura and Shimomura, 2002].

Trade economists have recently developed different trade models in which endogenous growth is generated either by the development of new varieties of intermediate or final goods or by the improvement of an existing set of goods with endogenous technologies [see, Chari and Hopenhayn, 1991; Martin and Ottaviano, 2001; Nocco, 2005; Brecher *et al.*, 2002; and Gustafsson and Segerstrom, 2010]. These studies attempted to formalize trade patterns with endogenous technological change and monopolistic competition. Within such frameworks the dynamic interdependence between trade patterns, R&D efforts, and various economic policies are connected. Nevertheless, most of these models do not explicitly deal with both capital and knowledge within the same framework. The model is this study is to integrate the two mainstreams of trade growth within a comprehensive framework. In fact, the model in this study is a further development of the two models by Zhang [1992, 2008a]. The previous studies were only concerned with examining equilibrium and comparative statics analysis without endogenous knowledge. This study simulates the model to show how the system moves over time with endogenous and how the motion of the system is affected when some parameters are changed. This paper is organized as follows. Section 2 defines the multi-country model with capital accumulation and knowledge creation. Section 3 shows that the dynamics of the world economy with J countries can be described by $(J+1)$ differential equations. As mathematical analysis of the system is too complicated, we demonstrate some of the dynamic properties by simulation when the world economy consists of three countries. Section 4 examines effects of changes in some parameters on the paths of the global economic dynamics. Section 5 concludes the study. The analytical results in Section 3 are proved in Appendix.

2. THE MULTI-COUNTRY TRADE MODEL WITH CAPITAL AND KNOWLEDGE

Each country has one production sector. Knowledge growth is through learning by doing. In describing the production sector, we follow the neoclassical trade framework. It is assumed that the countries produce a homogenous commodity. This follows the Oniki-Uzawa trade model and its various extensions with one capital goods [Ikeda and Ono ,1992]. Most aspects of production sectors in our model are similar to the neo-classical one-sector growth model [Burmeister and Dobell, 1970]. There is only one (durable) good in the global economy under consideration. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use capital and labor. Exchanges take place in perfectly competitive markets. Production sectors sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. All savings volunteered by households are absorbed by firms. We require savings and investment to be equal at any point of time. The system consists of multiple countries, indexed by $j = 1, ..., J$. Each country has a fixed labor force, $\overline{N}_j$, ($j = 1, ..., J$). Let prices be measured in terms of the commodity and the price of the commodity be unity.

We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the j th country. In the free trade system, the interest rate is identical throughout the world economy, i.e., $r(t) = r_j(t)$.

Behavior of producers

First, we describe behavior of the production sections. Let $F_j(t)$ and $K_j(t)$ stand for respectively the output level of the production sector and capital stocks employed by country j. We assume that there are three factors, knowledge, physical capital and labor at each point of time. The production functions are specified as follows

$$F_j(t) = A_j Z^{m_j}(t) K_j^{\alpha_j}(t) N_j^{\beta_j}(t), \ A_j > 0, \ \alpha_j + \beta_j = 1, \ \alpha_j, \beta_j > 0, \ j = 1, \cdots, J,$$

in which $Z(t)\,(> 0)$ is the world knowledge stock at time t. The total labor supply, $N_j(t)$, is

$$N_j(t) = T_j(t)\bar{N}_j,$$

where $T_j(t)$ is the working hours of a worker in country j. Here, we interpret $Z^{m_j/\beta_j}(t)$ as country j's level of human capital. The term $Z^{m_j/\beta}N_j$ is country j's human capital or qualified labor force. We see that the production function is a neoclassical one and homogeneous of degree one with the inputs. Here, we call m_j country j's knowledge utilization efficiency parameter. As cultures, political systems and educational and training systems vary between countries, m_j are different.

Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest, $r(t)$, and wage rates, $w_j(t)$, are determined by markets. Hence, for any individual firm $r(t)$ and $w_j(t)$ are given at each point of time. The production sector chooses the two variables, $K_j(t)$ and $N_j(t)$, to maximize its profit. The marginal conditions are given by

$$r + \delta_{kj} = A_j \alpha_j Z^{m_j} k_j^{-\beta_j}, \ w_j = A_j \beta_j Z^{m_j} k_j^{\alpha_j}, \tag{1}$$

where δ_{kj} are the depreciation rate of physical capital in country j and $k_j(t) \equiv K_j(t)/N_j(t)$.

Behavior of consumers

Each worker may get income from wealth ownership and wages. Consumers make decisions on consumption levels of goods as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang in the early 1990s. Let $\bar{k}_j(t)$ stand for the per capita wealth in country j. Each consumer of country j obtains income

$$y_j(t) = r(t)\bar{k}_j(t) + w_j(t)T_j(t), \ j = 1, \cdots, J, \tag{2}$$

from the interest payment $r\bar{k}_j$ and the wage payment $w_j T_j$. We call y_j the current income in the sense that it comes from consumers' payment and consumers' current earnings from ownership of wealth. The sum of income that consumers are using for consuming, saving, or transferring are not necessarily equal to the temporary income because consumers can sell wealth to pay, for instance, the current consumption if the temporary income is not sufficient for buying food and touring the country. The total value of wealth that a consumer in country j can sell to purchase goods and to save is equal to $\bar{k}_j(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income is equal to

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \tag{3}$$

The disposable income is used for saving and consumption. It should be noted that the value, $\bar{k}_j(t)$, (i.e., $p(t)\bar{k}_j(t)$ with $p(t)=1$), in the above equation is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we may consider $\bar{k}_j(t)$ as the amount of the income that the consumer obtains at time t by selling all of his wealth. Hence, at time t the consumer has the total amount of income equalling $\hat{y}_j(t)$ to distribute between consuming and saving. It should also be remarked that in the growth literature, for instance, in the Solow model, the saving is out of the current income, $y_j(t)$, while in this study the saving is out of the disposable income. This approach is discussed at length by Zhang (2008). Zhang has also examined the relations between his approach and the Solow growth theory, the Ramsey growth theory, the permanent income hypothesis, and the Keynesian consumption function in details.

The disposable income is used for saving and consumption. At each point of time, the representative consumer would distribute the total available budget among savings $s_j(t)$ and consumption of goods $c_j(t)$. The budget constraint is given by

$$c_j(t) + s_j(t) = \hat{y}_j(t). \tag{4}$$

Let T_0 denote the total available time. The time constraint requires that the amounts of time allocated to each specific use add up to the time available

$$T_j + \tilde{T}_j = T_0, \quad j = 1, \cdots, J. \tag{5}$$

Substituting (5) into the budget constraints (3) yields

$$w_j(t)\tilde{T}_j(t) + c_j(t) + s_j(t) = \bar{y}_j(t) \equiv (1 + r(t))\bar{k}_j(t) + T_0\, w_j(t). \tag{6}$$

We can interpret the right-hand term as the potential disposable income in the sense that if the worker devotes all the available time to work, how much he can use for consuming and saving. The term, $w_j \tilde{T}_j$, is the opportunity cost of leisure time. Hence, the left-hand is the sum of the opportunity cost, the expenditure of consumption and the saving. At each point of time, households decide the leisure time, consumption level and savings subject to the potential disposable income. We assume that utility level $U_j(t)$ is dependent on $\tilde{T}_j(t)$, $c_j(t)$ and $s_j(t)$ as follows

$$U_j(t) = \tilde{T}_j^{\sigma_{j0}}(t) c_j^{\xi_{j0}}(t) s_j^{\lambda_{j0}}(t), \quad \sigma_{j0}, \xi_{j0}, \lambda_{j0} > 0, \tag{7}$$

where σ_j, ξ_j and λ_j are respectively country 's propensities to use the leisure time, to consume goods, and to hold wealth. Maximizing subject to the budget constraints (6) yields

$$w_j(t)\tilde{T}_j(t) = \sigma_j \bar{y}_j(t), \quad c_j(t) = \xi_j \bar{y}_j(t), \quad s_j(t) = \lambda_j \bar{y}_j(t), \tag{8}$$

where

$$\sigma_j \equiv \rho_j \sigma_{j0}, \quad \xi_j \equiv \rho_j \xi_{j0}, \quad \lambda_j \equiv \rho_j \lambda_{j0}, \quad \rho_j = \frac{1}{\sigma_{j0} + \xi_{j0} + \lambda_{j0}},$$

where σ_{j0}, ξ_{0j} and λ_j are respectively household 's propensities to use the leisure time, to consume and to hold wealth. Here, for simplicity, we specify the utility function with the Cobb-

Douglas from. It would provide more insights if we take some other forms of utility functions. In this study we fix the preference structure.

According to the definitions of $s_j(t)$, the wealth accumulation of the representative household in country j is given by

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \qquad (9)$$

This equation simply states that the change in wealth is equal to the saving minus dissaving.

Knowledge creation through learning by doing

High creativity and wide application of knowledge is the main characteristic of modern economies. Like in the traditional growth models with endogenous, we assume that knowledge growth is through learning by doing. We propose the following equation for knowledge growth

$$\dot{Z}(t) = \sum_{j=1}^{J} \frac{\tau_j F_j(t)}{Z^{\varepsilon_j}(t)} - \delta_z Z(t), \qquad (10)$$

in which δ_z (≥ 0) is the depreciation rate of knowledge, and ε_j, and τ_j are parameters. The parameters τ_j and are non-negative. We interpret $\tau_j F / Z^{\varepsilon_j}$ as the contribution to knowledge accumulation through learning by doing by country $j's$ production sector. To see how learning by doing occurs, assume that knowledge is a function of country $j's$ total industrial output during some period

$$Z(t) = a_1 \left\{ \int_0^t F_j(\theta) d\theta \right\}^{a_2} + a_3$$

in which a_1, a_2 and a_3 are positive parameters. The above equation implies that the knowledge accumulation through learning by doing exhibits decreasing (increasing) returns to scale in the case of $a_2 < (>) 1$. We interpret a_1 and a_3 as the measurements of the efficiency of learning by doing by the production sector. Taking the derivatives of the equation yields $\dot{Z} = \tau_j F_j / Z^{\varepsilon_j}$, in which $\tau_j \equiv a_1 a_2$ and $\varepsilon_j \equiv 1 - a_2$.

Let $K(t)$ and $\bar{K}_j(t)$ stand for respectively the capital stocks of the world economy the capital stocks employed and the wealth owned by country j. The total capital stocks employed by the world is equal to the wealth owned by the world. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{k}_j(t) \bar{N}_j = \sum_{j=1}^{J} \bar{K}_j(t). \qquad (11)$$

The world production is equal to the world consumption and world net savings. That is

$$C(t) + S(t) - K(t) + \sum_{j=1}^{J} \delta_{kj} K_j(t) = F(t),$$

where

$$C(t) \equiv \sum_{j=1}^{J} c_j(t) \bar{N}_j, \quad S(t) \equiv \sum_{j=1}^{J} s_j(t) \bar{N}_j, \quad F(t) \equiv \sum_{j=1}^{J} F_j(t).$$

The trade balances of the economies are given by

$$E_j(t) = \left(\bar{K}_j(t) - K_j(t)\right) r(t).$$

When $E_j(t)$ is positive (negative), we say that country j is in trade surplus (deficit). When $E_j(t)$ is zero, country $j's$ trade is in balance.

We have thus built the model with trade, economic growth, capital accumulation, knowledge creation and utilization in the world economy in which the domestic markets of each country are perfectly competitive, international product and capital markets are freely mobile and labor is internationally immobile.

3. THE WORLD ECONOMIC DYNAMICS

As our model describe economic dynamics of multiple countries, the global dynamics should be described by highly dimensional differential equations. We will simulate the model when the households have different preferences. We first show that the dynamics of the world economy can be expressed by $(J+1)$ differential equations.

Lemma 1

The dynamics of the world economy is governed by the following $(J+1)$ – dimensional differential equations system with $Z(t)$, $k_1(t)$ and $\bar{k}_j(t)$, $j = 2, \cdots, J$, as the variables

$$\dot{Z} = \Lambda(k_1, Z),$$

$$\dot{k}_1 = \Lambda_1(k_1, \bar{k}_2, \ldots \bar{k}_J, Z),$$

$$\dot{\bar{k}}_j = \Lambda_j(k_1, \bar{k}_j, Z),$$

in which Λ_j and Λ are unique functions of Z, k_1 and $\bar{k}_j$ at any point of time, defined in Appendix. For given positive values of Z, k_1 and $\bar{k}_j$ at any point of time, the other variables are uniquely determined by the following procedure: $\bar{k}_1$ by (A7) $\rightarrow$ k_j, $j = 2, \cdots, J$ by (A1) $\rightarrow$ r and w_j by (A2) $\rightarrow \tilde{T}_j$ by (A3) $\rightarrow$ $T_j = T_0 - \tilde{T}_j \rightarrow$ $N_j = T_j \bar{N}_j$ $\rightarrow$ $K_j = k_j N_j$ $\rightarrow$ $\bar{y}_j$ by the definition $\rightarrow$ c_j and s_j by (8) $\rightarrow$ $F_j = A_j Z^{m_j} K_j^{\alpha_j} N_j^{\beta_j}$.

We have the dynamic equations for the world economy with any number of countries. The system is nonlinear and is of high dimension. It is difficult to generally analyze behavior of the system. We now solve equilibrium problem. For simplicity, we require $\delta_k = \delta_{kj}$, $j = 1, \ldots, J$. Equations (A1) and (A2) now become

$$k_j = \phi_j(k_1, Z) = \tau_{kj} Z^{\bar{m}_j} k_1^{\beta_1/\beta_j},$$

$$w_j = \bar{\phi}_j(k_1, Z) = \tau_{wj} Z^{m_{0j}} k_1^{\alpha_{wj}}, \quad j = 1, \cdots, J, \tag{12}$$

where

$$\tau_{kj} \equiv \left(\frac{A_j \alpha_j}{A_1 \alpha_1}\right)^{1/\beta_j}, \quad \bar{m}_j \equiv \frac{m_j - m_1}{\beta_j}, \quad \tau_{wj} \equiv A_j \beta_j, \quad m_{0j} \equiv m_j + \alpha_j \bar{m}_j, \quad \alpha_{wj} \equiv \frac{\beta_1 \alpha_j}{\beta_j}.$$

By equations (7), we have $s_j = \bar{k}_j$. By the definition of R and equations (1), we have

$$R(k_1, Z) = \lambda_1 \left(\lambda_{u1} - A_1 \alpha_1 Z^{m_1} k_1^{-\beta_1}\right), \tag{13}$$

From $s_j = \bar{k}_j$ and (A9), we have

$$\bar{k}_j = \frac{\tau_{wj}\, T_0\, Z^{m_{0j}}\, k_1^{\alpha_{wj}}}{\lambda_{uj} - \alpha_1 A_1 Z^{m_1} k_1^{-\beta_1}}, \quad j = 2, \ldots, J, \tag{14}$$

where we use (1) and $\lambda_{uj} \equiv 1/\lambda_j - 1 + \delta_k$. By equations (A13), at equilibrium we have

$$\Omega_k(k_1, Z) \equiv \lambda_1 T_0 w_1 - R\left[\psi(k_1, Z) - \sum_{j=2}^{J} \psi_j(k_1, Z)\bar{k}_j\right] = 0, \tag{15}$$

in which we use $\Lambda = \Lambda_j = 0$. Substituting $k_j = \tau_{kj} Z^{\bar{m}_j} k_1^{\beta_1/\beta_j}$ into (A8) and setting the resulted equation at equilibrium, we have

$$\Omega_Z(k_1, Z) \equiv \sum_{j=1}^{J} \tau_j\, \tau_{kj}^{\alpha_j}\, A_j N_j Z^{x_j}\, k_1^{\alpha_j \beta_1/\beta_j} - \delta_z = 0, \tag{16}$$

in which $x_j \equiv m_j - \varepsilon_j + \alpha_j \bar{m}_j - 1$. We see that two equations, $\Omega_k(k_1, Z) = 0$ and $\Omega_Z(k_1, Z) = 0$, contain two variables, k_1 and Z. The two equations determine equilibrium values of k_1 and Z.

By equations (13), we determine $\bar{k}_j$ for $j = 2, \ldots, J$. Following the procedure in Lemma 1, we determine all the other variables at equilibrium. We see that in order to solve equilibrium the main problem is to solve $\Omega_k(k_1, Z) = 0$ and $\Omega_Z(k_1, Z) = 0$, for $k_1 > 0$ and $Z > 0$. As we cannot explicitly solve the equilibrium values of k_1 and Z, we simulate the model to illustrate properties of the global economy. We specify the parameters as follows:

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 8 \end{pmatrix}, \ \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.8 \\ 0.7 \end{pmatrix}, \ \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.2 \\ 0.1 \end{pmatrix}, \ \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.04 \\ 0.02 \end{pmatrix}, \ \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.32 \\ 0.31 \end{pmatrix},$$

$$\begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.02 \\ 0.015 \end{pmatrix}, \ \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \end{pmatrix}, \ \begin{pmatrix} \xi_{01} \\ \xi_{02} \\ \xi_{0.3} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.25 \\ 0.3 \end{pmatrix}, \ \begin{pmatrix} \lambda_{01} \\ \lambda_{02} \\ \lambda_{03} \end{pmatrix} = \begin{pmatrix} 0.75 \\ 0.7 \\ 0.65 \end{pmatrix},$$

$$\sigma_{01} = \sigma_{02} = \sigma_{03} = 0.25, \ T_0 = 1, \ \delta_k = 0.05, \ \delta_Z = 0.04. \tag{17}$$

Country $1, 2$ and $3's$ populations are respectively $3, 4$ and 8. Country 3 has the largest population. Country $1, 2$ and $3's$ total productivities, A_j, are respectively $1, 0.8$ and 0.7. Country $1, 2$ and $3's$ utilization efficiency parameters, m_j, are respectively 0.4, 0.2 and 0.1. Country 1 utilizes knowledge mostly effectively; country 2 next and country 3 utilizes knowledge lest effectively. We call the three countries respectively as developed, industrializing, and underdeveloped economies (DE, IE, UE). We specify the values of the parameters, α_j, in the Cobb-Douglas productions approximately equal to 0.3. The value is often used in empirical studies [for instance, Miles and Scott, 2005; Abel *et al.*, 2007]. The DE's learning by doing parameter, τ_{i1}, is the highest among the countries. The returns to scale parameters in learning by doing, ε_{ij}, are all positive, which implies that knowledge exhibits decreasing returns to scale in learning by doing. The depreciation rates of physical capital and knowledge are specified respectively at 0.05 and 0.04. The DE's propensity to save is 0.75 and the UE's propensity to save is 0.65. The value of the IE's propensity is between the two other countries. The three economies have the equal propensity to enjoy leisure. We introduce country $j's$ returns to scale parameters for the production sector as follows:

$$x_j^* \equiv \frac{m_j}{\beta_j} - \varepsilon_j - 1, \quad j = 1, 2, 3,$$

which are respectively equal to, $-0.7, -1.09, -1.34,$ with the specified values of the parameters. We have, $x_{ij}^* < 0$ for all j. As no economy in the global economy exhibits decreasing returns to scale, it is expected that the dynamic system has a unique equilibrium point and it is stable. We now show that the dynamic system has a unique equilibrium point. Figure no. 1 plots the two equations, $\Omega_k(k_1, Z) = 0$ and $\Omega_Z(k_1, Z) = 0$, for $k_1 > 0$ and $Z > 0$. The solid lines represent $\Omega_k(k_1, Z) = 0$ and the dashed line stands for $\Omega_Z(k_1, Z) = 0$.

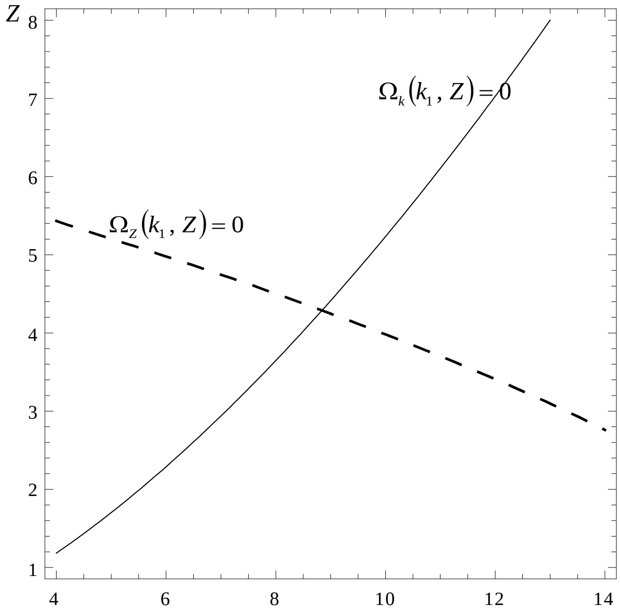

Figure no. 1 Solutions of Equations (15) and (16)

As shown in Figure 1, there is a unique meaningful solution as follows
$k_1 = 8.843, \quad Z = 4.288.$

The solution is meaningful in the sense that all the other variables are economically meaningful. We evaluate the other variables at the unique equilibrium point as in Table 1. The global output is 11.68 and the interest rate is about 6.7 percent. The shares of the global outputs by the DE, ID and UD are respectively 33, 27 and 40 percent. It should be noted that the population shares of the three economies are respectively 20, 26.7 and 53.3 percent. The per-worker output levels of the DE, ID and UD are respectively 3.44, 1.78 and 1.14. The differences in labor productivity are mainly due to the differences in knowledge utilization efficiency. The table also gives the labor and wealth distributions among the countries. Te capital stock employed per worker in the DE is almost twice as much as in the ID and almost three times in the UN. The wage rates in the DE, ID and UD are respectively 2.41, 1.30 and 0.91. The DE is in trade surplus and the other two economies in trade deficit.

The worker in the UD works longest hours and the worker in the DE shortest hours. The wealth and consumption per capita in the DE is much higher than in the UD.

Table no. 1 The equilibrium values of the global economy

Z	K	F	r	C	
4.29	30.95	11.68	0.067	10.80	
Country 1		Country 2		Country 3	
F_1	3.87	F_2	3.18	F_3	4.63
K_1	9.94	K_2	8.73	K_3	12.28
N_1	1.13	N_2	1.79	N_3	4.05
F_1 / N_1	3.44	F_2 / N_2	1.78	F_3 / N_3	1.14
k_1	8.82	k_2	4.87	k_3	3.03
$\bar{K}_1$	13.56	$\bar{K}_2$	8.04	$\bar{K}_3$	9.35
C_1	3.62	C_2	2.87	C_3	4.32
w_1	2.41	w_2	1.30	w_3	0.91
$\bar{k}_1$	4.52	$\bar{k}_2$	2.01	$\bar{k}_3$	1.17
c_1	1.21	c_2	0.72	c_3	0.54
T_1	0.375	T_2	0.448	T_3	0.507
E_1	0.24	E_2	-0.05	E_3	-0.19

So far we have been concerned with equilibrium. As the four eigenvalues at equilibrium are as follows

$$-0.376, \ -0.335, \ -0.130, \ -0.334,$$

we see that the equilibrium is stable. We start with different initial states not far away from the equilibrium point and find that the system approaches to the equilibrium point. This also demonstrates that the system is stable. In Figure 2, we plot the motion of the system with the following initial conditions

$$k_1(0) = 8.8, \ \bar{k}_2(0) = 1.6, \ \bar{k}_3(0) = 0.8, \ Z(0) = 3.$$

The system approaches to its equilibrium in the long term.

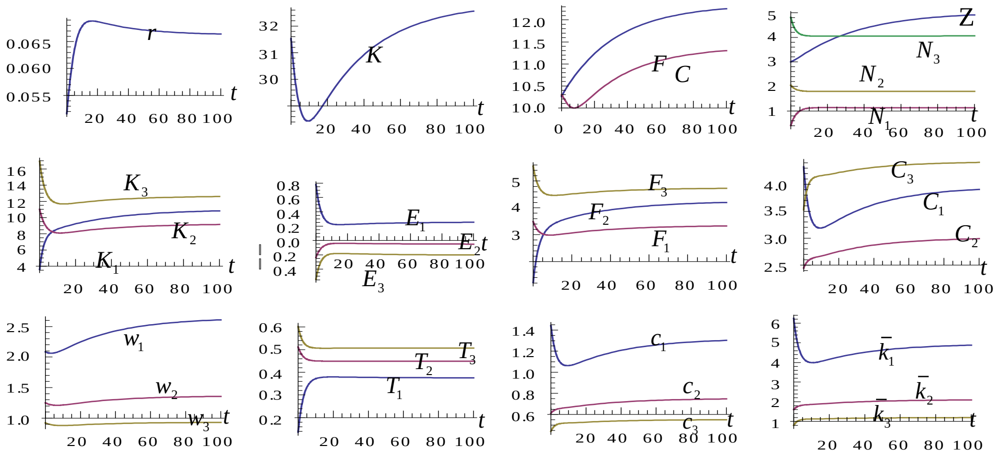

Figure no. 2 The Motion of the Global Economy

4. COMPARATIVE DYNAMIC ANALYSIS

We simulated the motion of the dynamic system. It is important to ask questions such as how a developing economy like India or China may affect the global economy as its technology is improved or population is enlarged; or how the global trade patterns may be affected as technologies are further improved or propensities to save are increased in developed economies like the USA or Japan. The rest of this paper examines effects of changes in some parameters on dynamic processes of the global economic system. First, we examine the case that all the parameters, except country 1's knowledge utilization efficiency, m_1, are the same as in (17). We reduce the knowledge efficiency parameter, m_1, from 0.4 to 0.38. The simulation results are demonstrated in Figure 3. In the plots, a variable $\bar{\Delta} x_j(t)$ stands for the change rate of the variable, $x_j(t)$, in percentage due to changes in the parameter value from m_{10} $(= 0.4$ in this case) to m_1 $(= 0.38)$. That is

$$\bar{\Delta} x_j(t) \equiv \frac{x_j(t; m_1) - x_j(t; m_{10})}{x_j(t; m_{10})} \times 100,$$

where $x_j(t; m_1)$ stands for the value of the variable x_j with the parameter value m_1 at time t and $x_j(t; m_{10})$ stands for the value of the variable x_j with the parameter value m_{10} at time t. We will use the symbol $\bar{\Delta}$ with the same meaning when we analyze other parameters.

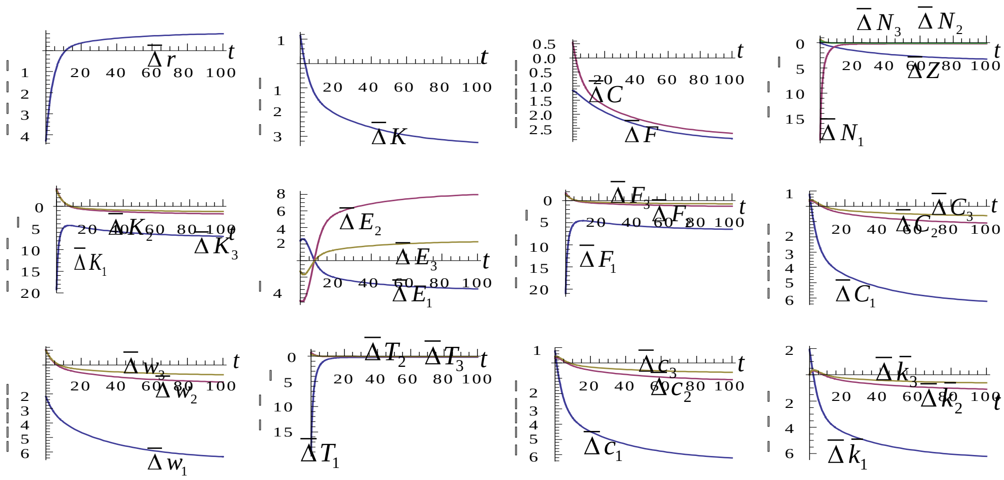

Figure no. 3 The developed economy applies knowledge less effectively

As the DE uses knowledge less effectively, the knowledge, output and capital of the global economy are reduced. The rate of interest falls initially and then rises (over its previous level). In the opposite direction to the motion of the rate of interest, the consumption level of each country rises initially and then falls. In the long term the labor supply of each country is not affected. The capitals stocks employed by all the countries are reduced. The wage rates and per capita wealth of all the countries are reduced. We see that all the economies in the well-connected global system will suffer from a reduction in the DE's knowledge utilization.

We now examine effects of changes in creativity parameters in learning by doing. We increase the DE's learning by doing efficiency as follows: $\tau_1 : 0.03 \Rightarrow 0.04$. The effects are plotted in Figure 4. The knowledge, global wealth and output levels are all increased. The rate of interest rises initially and falls later on. The total output and consumption levels, total wealth, per capita consumption levels, and per capita wealth levels of the three economies are all increased. The trade balance of the DE improves and the other two economies deteriorate. All the households in the global economy work longer hours than before. The DE benefits more than the other two economies because its knowledge utilization efficiency is higher than the other economies.

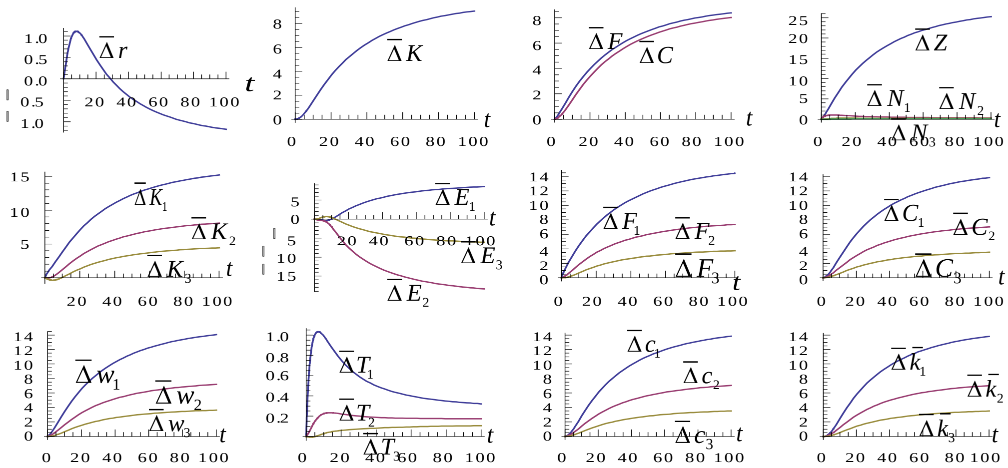

Figure no. 4 The developed economy becomes more creative

5. CONCLUSIONS

This paper built a multi-country growth model of endogenous labor supply with capital accumulation and knowledge creation with learning by doing. If we neglect international trade and assume knowledge stock and working time to be constant, then the model is similar to the Solow growth model. The trade model without constant knowledge stock is similar to the Oniki-Uzawa model. We show that the dynamics of J -country world economy is described by $(J+1)$ differential equations. We simulated the motion of the model and demonstrated effects of changes in the parameters. The model with the specified parameter values (which imply no-increasing return scales) has a unique equilibrium. It can be shown that if some economies exhibit increasing returns to scale, then there exist multiple equilibrium points. To which equilibrium point the system evolves is dependent on initial conditions. When differences in knowledge utilization efficiency and preference exist, our model does not generate global convergence. It is well known that one-sector growth model has been generalized and extended in many directions. There are multiple production sectors and households are not homogenous. It is not difficult to generalize our model along these lines. It is also possible to develop the model in discrete time. We may analyze behavior of the model with other forms of production or utility functions.

Appendix: Proving Lemma 1

First, from equations (1) we obtain

$$k_j = \phi_j(k_1, Z) \equiv \left(\frac{A_j \alpha_j Z^{m_j}}{A_1 \alpha_1 Z^{m_1} k_1^{-\beta_1} + \delta_j} \right)^{1/\beta_j}, \quad j = 1, \ldots, J, \tag{A1}$$

where $\delta_j \equiv \delta_{k1} - \delta_{kj}$. It should be noted that $\phi_1 = k_1$. From equations (1) and (A1), we determine the wage rates as functions of $k_1(t)$ and $Z(t)$ as follows

$$r = \bar{\phi}(k_1, Z) \equiv A_1 \alpha_1 Z^{m_1} k_1^{-\beta_1} - \delta_{k1}, \quad w_j = \bar{\phi}_j(k_1, Z) \equiv A_j \beta_j Z^{m_j} \phi_j^{\alpha_j}(k_1, Z), \quad j = 1, \cdots, J. \tag{A2}$$

From $w_j \tilde{T}_j = \sigma_j \bar{y}_j$ in (8) and the definition of $\bar{y}_j$, we have

$$\tilde{T}_j = \left(\frac{1 + \bar{\phi}(k_1, Z)}{\bar{\phi}_j(k_1, Z).} \right) \sigma_j \bar{k}_j + \sigma_j T_0 . \tag{A3}$$

By $N_j = T_j \bar{N}_j$, (5) and (A3), the labor supply is given by

$$N_j = (1 - \sigma_j) \bar{N}_j T_0 - \left(\frac{1 + \bar{\phi}(k_1, Z)}{\bar{\phi}_j(k_1, Z)} \right) \sigma_j \bar{N}_j \bar{k}_j . \tag{A4}$$

From $K_j = k_j N_j$ and (A4), we solve

$$K_j = (1 - \sigma_j) \bar{N}_j T_0 \phi_j(k_1, Z) - \tilde{\phi}_{mj}(k_1, Z) \bar{k}_j , \tag{A5}$$

where

$$\tilde{\phi}_{mj}(k_1, Z) \equiv \left(\frac{1 + \bar{\phi}(k_1, Z)}{\bar{\phi}_j(k_1, Z).} \right) \phi_j(k_1, Z) \sigma_j \bar{N}_j .$$

By (11) and (A5), we have

$$K = \tilde{\phi}_0(k_1, Z) - \sum_{j=1}^{J} \tilde{\phi}_{mj}(k_1, Z) \bar{k}_j , \tag{A6}$$

where

$$\tilde{\phi}_0(k_1, Z) \equiv \sum_{j=1}^{J} (1 - \sigma_j) \bar{N}_j T_0 \phi_j(k_1, Z).$$

From (A6) and (11), we have

$$\bar{k}_1 = \tilde{\psi}(k_1, Z, \{\bar{k}_j\}) \equiv \psi(k_1, Z) - \sum_{j=2}^{J} \psi_j(k_1, Z) \bar{k}_j , \tag{A7}$$

in which

$$\psi(k_1, Z) \equiv \frac{\tilde{\phi}_0(k_1, Z)}{\bar{N}_1 + \tilde{\phi}_{m1}(k_1, Z)}, \quad \psi_j(k_1, Z) \equiv \frac{\bar{N}_j + \tilde{\phi}_{mj}(k_1, Z)}{\bar{N}_1 + \tilde{\phi}_{m1}(k_1, Z)}.$$

From (A5)-(A7) we see that the capital distribution among the countries and the global capital are uniquely determined as functions of $k_1(t)$ and $Z(t)$. Substituting $F_j = Z^{m_j} K_j^{\alpha_j} N_j^{\beta_j}$ into (10), we have

$$\dot{Z} = \Lambda(k_1, Z) \equiv \sum_{j=1}^{J} \tau_j A_j N_j^{\beta_j} Z^{m_j - \varepsilon_j} K_j^{\alpha_j} - \delta_z Z . \tag{A8}$$

We see that the motion of Z can be described as a unique function of k_1 and Z.

Introduce $\{\bar{k}(t)\} \equiv (\bar{k}_2(t), \cdots, \bar{k}_J(t))$. From (A7) we see that country 1's per capita wealth, $\bar{k}_1(t)$, can be expressed as a unique function of the knowledge, country 1's capital intensity and the other countries' per capita wealth, $\{\bar{k}(t)\}$, at any point of time. Substituting $\bar{y}_j = (1 + r)\bar{k}_j + T_0 w_j$ into $s_j = \lambda_j \bar{y}_j$ yields

$$s_j = (1 + r)\lambda_j \bar{k}_j + \lambda_j T_0 w_j . \tag{A9}$$

Substituting (A9) into (7), we have

$$\dot{\bar{k}}_1 = \lambda_1 T_0 w_1 - R(k_1, Z) \bar{k}_1 , \tag{A10}$$

$$\dot{\bar{k}}_j = \Lambda_j(k_1, \bar{k}_j, Z) \equiv \lambda_j T_0 w_j - (1 - \lambda_j - \lambda_j r) \bar{k}_j , \quad j = 2, ..., J , \tag{A11}$$

in which $R(k_1, Z) \equiv 1 - \lambda_1 - \lambda_1 r$. Taking derivatives of equation (A7) with respect to t yields

$$\dot{\bar{k}}_1 = \tilde{\psi}_1 \dot{k}_1 + \tilde{\psi}_2 \dot{Z} - \sum_{j=2}^{J} \psi_j (k_1, Z) \dot{\bar{k}}_j, \tag{A12}$$

where

$$\tilde{\psi}_1 \equiv \frac{\partial \psi}{\partial k_1} - \sum_{j=2}^{J} \frac{\partial \psi_j}{\partial k_1} \bar{k}_j, \quad \tilde{\psi}_2 \equiv \frac{\partial \psi}{\partial Z} - \sum_{j=2}^{J} \frac{\partial \psi_j}{\partial Z} \bar{k}_j.$$

Equaling the right-hand sizes of equations (A10) and (A12), we get

$$\tilde{\psi}_1 \dot{k}_1 + \tilde{\psi}_2 \dot{Z} - \sum_{j=2}^{J} \psi_j (k_1, Z) \dot{\bar{k}}_j = \lambda_1 T_0 w_1 - R \tilde{\psi}(k_1, Z).$$

Substitute equation (A8) into the above equation

$$\dot{k}_1 = \Lambda_1 \left(k_1, \{\bar{k}_j\}, Z\right) \equiv \left[\sum_{j=2}^{J} \psi_j \Lambda_j + \lambda_1 T_0 w_1 - R \tilde{\psi} - \tilde{\psi}_2 \Lambda\right] \frac{1}{\tilde{\psi}_1}, \tag{A13}$$

where we use equations (A4) and (A3). In summary, we proved Lemma 1.

References

[1] Abel, A., Bernanke, B.S., Croushore, D., *Macroeconomics*, Prentice Hall, New Jersey, 2007.
[2] Agénor, P.R., Does globalization hurt the poor?, *International Economics and Economic Policy*, 1, 21-51, 2004.
[3] Bhagwati, J.N., (Ed.) *International trade - selected readings*, MIT Press, Mass., Cambridge, 1991.
[4] Blum, B.S., Trade, technology, and the rise of the service sector: the effects on us wage inequality, *Journal of International Economics*, 74, 441-58, 2008.
[5] Brecher, R.A., Chen, Z.Q., Choudhri, E.U., Absolute and comparative advantage, reconsidered: the pattern of international trade with optimal saving, *Review of International Economics*, 10, 645-656, 2002.
[6] Burmeister, E., Dobell, A.R., *Mathematical theories of economic growth*, Collier Macmillan Publishers, London, 1970.
[7] Chari, V.V., Hopenhayn, H., Vintage human capital, growth, and the diffusion of new technology, *The Journal of Political Economy*, 99, 1142-65, 1991.
[8] Ehrlich, I., Lui, F., The problem of population and growth: a review of the literature from Malthus to contemporary models of endogenous population and endogenous growth, *Journal of Economic Dynamics and Control*, 21, 205-4, 1997.
[9] Findlay, R., Growth and development in trade models, in Jones, R.W., Kenen, R.B. (Eds.): *Handbook of international economics*, North-Holland, Amsterdam, 1984.
[10] Frenkel, J.A., Razin, A., *Fiscal policy and the world economy*, MIT Press, MA., Cambridge, 1987.
[11] Gustafsson, P., Segerstrom, P., North-south trade with increasing product variety, *Journal of Development Economics*, 92, 97-106, 2010.
[12] Ikeda, S., Ono, Y., Macroeconomic dynamics in a multi-country economy - a dynamic optimization approach, *International Economic Review*, 33, 629-644, 1992.
[13] Jensen, B.S., *The dynamic systems of basic economic growth models*, Kluwer Academic Publishers, Dordrecht, 1994.
[14] Jensen, B.S., Wong, K.Y. (Eds.), *Dynamics, economic growth, and international trade*, The University of Michigan Press, Ann Arbor, 1998.
[15] Kemp, M.C., Foreign investment and national advantage, *Economic Record*, 28, 56-62, 1961.
[16] Krugman, P., Venables, A.J., Globalization and the Inequality. *Quarterly Journal of Economics* 110, 857-80, 1995.

[17] MacDougall, G.D.A., The benefits and costs of private investment from abroad: a theoretical approach, *Economic Record*, 27, 13-15, 1960.
[18] Manasse, P., Turrini, A., Trade, wages, and ‘superstars’, *Journal of International Economics*, 54, 97-117, 2001.
[19] Martin, P., Ottaviano, G., Growth and agglomeration, *International Economic Review*, 42, 947-68, 2001.
[20] Madsen, J.B., Technology Spillover through Trade and TFP Convergence: 135Years of Evidence for the OECD Countries, *Journal of International Economics* 72, 464-80, 2007.
[21] Miles, D., Scott, A., *Macroeconomics - Understanding the Wealth o Nations*, John Wiley Sons, Ltd., Chichester, 2005
[22] Nakajima, T., Catch-up in turn in a multi-country international trade model with learning-by-doing and intervention, *Journal of Development Economics*, 72, 117-38, 2003.
[23] Nishimura, K., Shimomura, K., Trade and indeterminacy in a dynamic general equilibrium model, *Journal of Economic Theory*, 105, 244-60, 2002.
[24] Nocco, A., The rise and fall of regional inequalities with technological differences and knowledge spillovers, *Regional Science and Urban Economics*, 35, 542-69, 2005.
[25] Obstfeld, M., Rogoff, K., *Foundations of International Macroeconomics*, MIT Press, Mass., Cambridge, 1998.
[26] Oniki, H., Uzawa, H., Patterns of trade and investment in a dynamic model of international trade, *Review of Economic Studies*, 32, 15-38, 1965.
[27] Sorger, G., On the multi-country version of the Solow-Swan model, *The Japanese Economic Review*, 54, 146-64, 2002.
[28] Valdés, B., *Economic Growth - Theory, Empirics and Policy*, Edward Elgar, Cheltenham, 1999.
[29] Vellutini, C., Capital mobility and underdevelopment traps, *Journal of Development Economics*, 71, 435–62, 2003.
[30] Wong, K.Y., *International Trade in Goods and Factor Mobility*, MIT Press, Mass., Cambridge, 1995.
[31] Zhang, W.B., Trade and world economic growth - differences in knowledge utilization and creativity, *Economic Letters*, 39, 199-206, 1992.
[32] Zhang, W.B. *International trade theory: capital, knowledge, economic structure, money and prices over time and space*, Springer, Berlin, 2008.
[33] Zhang, W.B., A multi-region economic model with migration, housing and regional amenity, *Annals of the Alexandru Ioan Cuza University of Iasi*, Economic Sciences Section, 2008, 321-50, 2008a.