FISCAL FEDERALISM AND PUBLIC DEBT FISCAL INSTABILITY DUE TO MULTI LEVEL BORROWING IN A MODEL OF NEOCLASSICAL GROWTH

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Abstract

A neoclassical growth model with constant saving rates, publicly provided services and public debt is used to analyse the impact of a federal structure. The main focus is on the existence and the stability of steady states. It is shown that the equilibrium in a federation with publicly provided services is more likely to be unstable than the equilibrium with public capital as a substitute. Moreover, only dynamic inefficient solutions are stable. Due to additional instability, the need for flexible vertical assignment of public functions is further stressed.

Keywords: Public debt, fiscal federalism, growth, stability
JEL classification: E13, E62, H62, H74

1. INTRODUCTION

An ongoing discussion concerning the further development of the European Union (EU) is the question how to finance the common activities at this level. In this context, the actual discussion mainly focuses on the existing financial framework and revenue policy. Since an increasing part of the EU expenditures are related to investments, the discussion about a borrowing competence at the community level could also be recommended. Therefore, following the "pay-as-you-use-principle", a borrowing competence would be an adequate measure to redistribute the burden to the (prospective) members who profit from the actual expenditure policy. This question of an independent borrowing competence is not limited to the EU as a supra national level. The problem of an additional borrowing competence is also relevant at the state/local. Thereby, [Wildasin, 2004] raises the question, "if the national government is able to borrow, does it really matter whether subnational governments are also able to borrow?" Hence, the question of an independent borrowing competence is also relevant for transition countries being in process to design their federal structure. An example is Albania which has introduced a sub national borrowing competence in 2008 [Law nr.9869, dated 4 February, 2008].

As shown by [Wenzel and Wrede, 2000] in an exogenous growth model, public debt in a federation can lead to instability of steady states. This instability implies that the fiscal adjustments at one level have a destabilizing effect on the public budget at the other level. Hence, the arising fiscal externality forces the jurisdictions to adjust their measures more of-

ten leading to an unstable tax and expenditure policy. Although a jurisdiction has not changed its fiscal policy, its public debt may rise dramatically due to a change in the policy of the other level. Especially in a context of limited tax instruments (e.g. budget restraint of the EU or local taxes for communities) this may lead to over borrowing. Contrary to [Ahmad, Albino-War and Singh, 2005], this problem of instability is not a question of overall aggregate exposure to risk. The problem of instability arises due to the additional borrowing competence at a second level beside of the national level.

Since the model of [Wenzel and Wrede, 2000] is based on the assumption that public and private investments are perfect substitutes, the question arises if the result also holds for public productive activity as a separated argument in the production function. In case of a separate activity, the marginal product of capital increases with an increase of the public activity, while in the case of perfect substitutes a decrease arises.

This paper investigates within the given structure by [Wenzel and Wrede, 2000] if a congested productive governmental input leads to fundamental changes of the results. The paper is organized in the following manner. In the second section, I will introduce the model. In section three, the stability of equilibria in a centralized state will be analyzed. The results will be also used as reference point for the examination of the federal model in the fourth section. It will be shown that, against common results in the growth theory, income taxation is no longer an appropriate measure to internalize the effects of impure public goods in a federation with public debt.

The upcoming chapter introduces the model structure. Chapter 3 analyses the reference case of a centralized state while chapter 4 focuses on the decentralized setting of different jurisdiction levels. Chapter 5 concludes the paper.

2. THE MODEL

2.1. Production

The aggregate production Y in the closed economy is determined by the stock of capital K, assumed to be infinitely durable, N the quantity of labor and publicly provided services G^p. Differently to [Wenzel and Wrede, 2000], public productive activity is not interpreted as a perfect substitute to private investment. In return, G^p denotes the productive government expenditures, which are introduced as a flow. One can argue that productive state activity, as infrastructure, should be more appropriately introduced as a stock. However, the resulting additional dynamic equation would only increase the complexity of the model without having a relevant influence on existence and stability of a steady state.[1] Furthermore, I assume that the public good is not a pure public good in the original [Samuelson, 1954] sense. Meanwhile government services are rival and not excludable, they are subject to congestion. For a given level of public services $\bar{G}^p$, the quantity available for the individual production is tied to the aggregate output.

Therefore, the aggregate output is determined by the following Cobb-Douglas production function:

$$Y = N^{1-\alpha} K^{\alpha} \left(\frac{G^p}{Y} \right)^{\beta} \text{ with } \alpha, \beta > 0 . \tag{1}$$

Since I want to focus only on vertical aspects, I assume that the regions in the economy are identical and that capital and labor are mobile factors within the economy.

As the production function is linearly homogeneous, it can be expressed in intensive form as

$$f\left(k, g^p\right) := y = Y/AN = k^\alpha \left(g^p\right)^\beta, \text{ with } k := \frac{K}{N},\ g^p := \frac{G^p}{Y}. \tag{2}$$

As the firms do not realize the congestion, they regard the ratio of productive public service to gross output as given $\bar{g}^p$. Thus, profit maximization implies that the private marginal product r exceeds the (social) marginal product of capital, i.e. that[2]

$$r := \frac{\partial f\left(k, \bar{g}^p\right)}{\partial k} = \alpha \frac{f\left(k, \bar{g}^p\right)}{k}, \tag{3}$$

$$f_k(k, g^p) := \frac{\partial f\left(k, g^p\right)}{\partial k} = \frac{\alpha}{1+\beta} \frac{f\left(k, g^p\right)}{k} \neq r. \tag{4}$$

In combination with the same assumption for the wages, it follows that profit is zero.

2.2. State

I suppose two levels of government, the state level and the federal level (indicated by the subscripts s and f). Variables concerning public activity without a subscript indicate the aggregated variable. Each level has its own tax revenues out of a flat-rate tax on both sources of income, output and interest payments on public bonds[3] rB (total income tax), with a constant tax rate $\tau_i > 0$.

$$T_i = \tau_i \left(Y + rB\right), \quad B = B_s + B_f, \quad i = s, f. \tag{5}$$

The following variables stay also time-invariant: the government expenditures to output ratios $G_i/Y = g_i$, the shares of productive expenditures κ_i, with $0 < \kappa_i < 1$, and consumptive purchases $(1-\kappa_i)$.

$$G^p = \kappa_s G_s + \kappa_f G_f =: \kappa G. \tag{6}$$

Equation (6) implies that the publicly provided services of the federal and the state level are perfect substitutes.

As mentioned in the context of the production function, I assume a perfect symmetry on the state level. Therefore, I set an identical tax rate, an identical expenditure policy concerning the expenditure-output ratio and ratio of productive to consumptive purchases for all states. Thus, production remains independent of the regional location.

Finally, I assume that a budget deficit D_i is financed completely by public borrowing. Hence, the sum of primary deficit $\left(G_i - T_i\right)$ and interest payments rB_i has to be equal to the variation of public debt $\dot{B}_i$[4]. Each level of government faces the following dynamic government budget restraint (GBR):

$$\dot{B}_i = G_i - T_i + rB_i = \left(g_i - \tau_i\right)Y - \tau_i rB + rB_i \quad i = s, f. \tag{7}$$

2.1. Households

Referring to [Solow, 1956], I consider a time-invariant saving rate s of disposable income. In the presented closed economy without depreciation, the increase of capital is equal to private saving reduced of the global public deficit D.

$$\dot{K} = s(1-\tau)\left(Y + rB\right) - \dot{B}, \quad 0 < s < 1, \tag{8}$$

with $B = B_s + B_f$, $\dot{B} = \dot{B}_s + \dot{B}_f$, $\tau = \tau_s + \tau_f$.

For n being the exogenously given rate of labor growth, the model can be described by and analyzed with the following three differential equations:

$$\dot{k} = E^1\left(k,b\right), \tag{9a}$$

$$\dot{b}_s = E^2\left(k,b,b_s\right), \tag{9b}$$

$$\dot{b}_f = E^3\left(k,b,b_f\right), \tag{9c}$$

whereby

$$E^1(\bullet) \equiv \left(1-(1-s)(1-\tau)-g\right) f(k,g^p) - (1-s)(1-\tau)rb - nk,$$

$$E^2(\bullet) \equiv \left(g_s - \tau_s\right) f(k,g^p) + (r-n)b_s - \tau_s rb,$$

$$E^3(\bullet) \equiv \left(g_f - \tau_f\right) f(k,g^p) + (r-n)b_f - \tau_f rb,$$

with $r = (1+\beta) f_k(k,g^p)$ and $b = b_s + b_f$.

The first equation presents the fundamental equation of capital accumulation, which is not determinate by the vertical division of public debt. The second and third equations describe the development of the state respectively the federal debt.

For the purpose of a dynamic equation of total debt that is independent of the debt structure, (9b - 9c) are aggregated to (9b'). In addition, the dynamic of the vertical division of debt is examined in equation (9c'), based on the federal- to total-debt ratio $\lambda = b_f / b$ respectively its development: $\dot{\lambda} = \left(\dot{b}_f - \dot{b}\lambda\right)/b$.

$$\dot{k} = \tilde{E}^1\left(k,b\right), \tag{9a'}$$

$$\dot{b} = \tilde{E}^2\left(k,b\right), \tag{9b'}$$

$$\dot{\lambda} = \tilde{E}^3\left(k,b,\lambda\right), \tag{9c'}$$

where

$$\tilde{E}^1(\bullet) \equiv \left(1-(1-s)(1-\tau)-g\right) f\left(k,g^p\right) - (1-s)(1-\tau)rb - nk,$$

$$\tilde{E}^2(\bullet) \equiv (g-\tau) f\left(k,g^p\right) + (1-\tau)rb - nb,$$

$$\tilde{E}^3(\bullet) \equiv \left(g_f - \tau_f - (g-\tau)\lambda\right) f\left(k,g^p\right)/b + \left(\tau\lambda - \tau_f\right)r,$$

with $r = (1+\beta) f_k(k,g^p)$ and $b = b_s + b_f$.

Representing also the centralized state, the economic growth system (9a'-9b') is independent of the vertical distribution of public debt. Therefore, its dynamics and steady states can be analyzed using a geometrical approach.

3. DYNAMICS AND STEADY STATE IN A CENTRALIZED STATE

In a steady state, neither capital nor aggregate public debt changes:

$$\tilde{E}^1\left(k,b\right) = 0, \tag{10a}$$

$$\tilde{E}^2\left(k,b\right) = 0. \tag{10b}$$

Hence, the equations for the isoclines are:

$$kk: \quad b(k) = \frac{\left[1-(1-s)(1-\tau)-g\right] f\left(k, g^{p}\right) - nk}{(1-s)(1-\tau)(1+\beta)\, f_k\left(k, g^{p}\right)} \tag{11a}$$

$$bb: \quad b(k) = \frac{(g-\tau)\, f(k, g^{p})}{n-(1-\tau)(1+\beta)\, f_k\left(k, g^{p}\right)}. \tag{11b}$$

Geometrically, a steady state is the intersection of the isoclines in the b-k phase diagram. Therefore, the stability characteristics of a steady state can be described through both the shapes and intersections of the isoclines and the shapes of the solution trajectories specified by the dynamics of the system (9a' – 9b').[5] Given the bb-isocline (11b), it follows immediately that numerator and denominator must have the same sign for a steady state with positive public debt.

Proposition 1: In a centralized state there is no asymptotically stable steady state with a tax rate higher than the expenditure to output ratio.

Thus I can focus the geometrical analyze on the case of $g > \tau$.

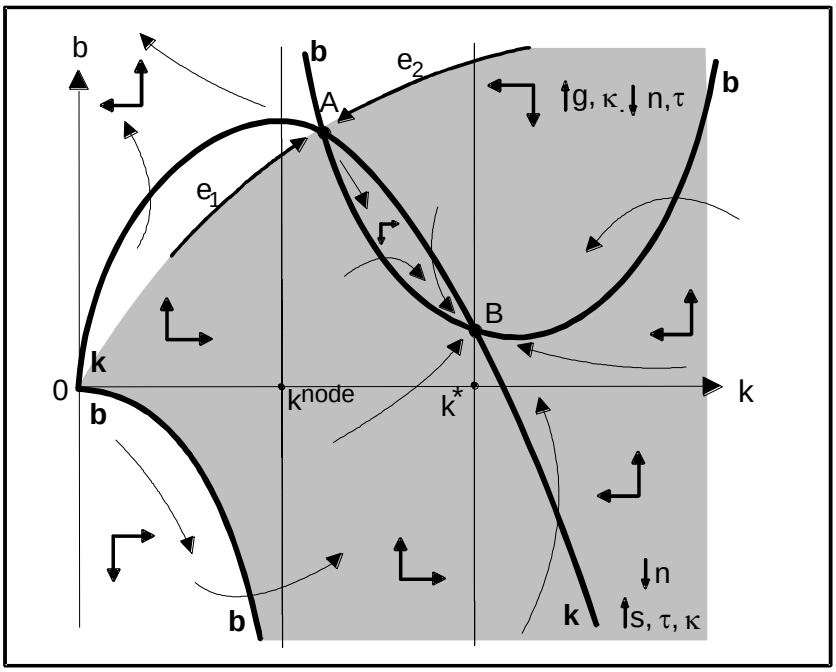

Figure no. 1 Steady states and Stability in the Centralized State and $g > \tau$

Figure no.1 shows the case in which the total tax rate is lower than the total expenditure to output ratio. In this constellation, two steady states with positive public debt exist. Further, the one with the higher capital intensity is an unambiguously stable steady state.

Proposition 2: In a centralized state, a steady state with positive public debt and a tax rate smaller than the expenditure-output ratio will be:

a.a saddle point, iff the slope of the kk-isokline exceeds the slope of the bb-isocline, and it will be

b. (locally) asymptotically stable iff the slope of the bb-isocline exceeds the slope of the kk-isocline and if simultaneously the slope of the kk-isocline is lower than $\tilde{E}_b^1 / \tilde{E}_b^2 > 0$.

Compared with the case of perfect capital substitutes, the following aspects are remarkable. The variation of g has no longer a definite negative effect on the kk-isocline. Now it depends on which effect dominates: the negative finance-effect or the positive output-effect. Furthermore, the optimal share of productive public services is pre-determinate by the production function with $g^p = \beta/1+\beta$. This follows out of the optimality condition of $f_{g^p}\left(k, g^p\right) = 1$. Finally, the tax rate on capital income must be sufficiently high so that the net interest rate $\left(1-\tau\right)r$ is lower than the social marginal product of capital. Otherwise the steady state will be dynamic inefficient or unstable. This follows directly from Equation (11b) with the necessity of a positive denominator and the Golden rule identity $f_k\left(k^*, g^p\right) = n$.

4. DYNAMICS AND STEADY STATE IN A FEDERATION

Additional to the economical equilibrium (10a-b), a steady state in a federation requires:

$$\tilde{E}^3\left(\bullet\right) = 0 . \tag{12}$$

Equation (12) can be seen as a political equilibrium since it presents a constant ratio of state- to total-debt and therefore a constant ratio of federal- to total-debt. If Equation (12) does not hold, new assignments of tax rates and/or expenditure competence are needed in the long run.

Proposition 3: In the (linearized) federal system, only a dynamically inefficient steady state can be locally asymptotically stable.

Proposition 4: In the federation, a steady state can only be locally asymptotic stable iff it is also stable in the centralized system alone.

While the propositions 3 and 4 remain unchanged, compared with the case of perfect capital substitutes, the necessary condition of Proposition 3 has changed. In the case of perfect substitution of private and public capital, the necessary condition for a stable solution in federation is [Wenzel and Wrede 2000: 103]:

$$\frac{\partial \tilde{E}^3}{\partial \lambda} = f_k\left(k\right) - n < 0 . \tag{13}$$

In the case of congested public services, the condition is modified to:

$$\frac{\partial \tilde{E}^3}{\partial \lambda} = \left(1+\beta\right) f_k\left(k, g^p\right) - n < 0 \tag{14}$$

Hence, I can conclude:

Proposition 5: The assumption of productive impure public services increases the dynamic inefficiency of a stable solution compared with the case of perfect capital substitutes.

In the context of this federal growth model, the taxation of capital income cannot be an appropriate measure of internalization. Since the interest rate on public bonds is equal to the private marginal profit rate, the interdependence between the levels of governments increases due to the introduction of the impure public services. Let us assume e.g. that the federal level increases its government-output ratio and at the same time its tax rate so that the net private profit rate of capital do not change as well. Thus, the net payments of interest on bonds stay also unchanged. Furthermore, I assume that the amount of additional tax revenues is by chance the same as the increased expenditures. In the case of the centralized state, the model persists at the same equilibrium values. In the case of the federal state, the ratio of state to federal net payments of interest on bonds has changed fundamentally. While the net payments of interest for the federal level remain constant, the state level has to pay a higher net interest rate $(1-\tau_s)rB$ which leads to an increasing debt.

Concerning the Golden rule solution, I propose a divergent interpretation to [Wenzel and Wrede, 2000: 104]. Using (9c'), (12) can be realized by the simultaneous fulfillment of:

$$g_f - \tau_f - \lambda(g-\tau) = 0 \text{ and } \tau\lambda - \tau_f = 0 \tag{15}$$

which leads to the condition

$$\frac{g_f - \tau_f}{g-\tau} = \frac{\tau_f}{\tau} \Leftrightarrow \frac{g_f}{\tau_f} = \frac{g}{\tau}. \tag{16}$$

Hence, in the case of such a political symmetry that the expenditure tax ratios over all levels are identical, the dynamic and stability of the system is only determined by the basic central system (9a'-9b').

5. CONCLUSION

I can resume that the main results of [Wenzel and Wrede, 2000], concerning the stability of equilibrium in the federal model, are also consistent with the assumption of public services, which are subject to congestion. Moreover, under the new assumptions the possibility of instable steady states increases and the usual solution of taking income taxes as an instrument for internalization fail in the given context.

Based on the actual assignments of expenditure and finance competence of the European Union, the existing inflexibility forces us to reject a debt competence for the supranational level. Regarding the competence of revenues, the EU is completely dependent on the willingness of the member states. If a debt competence is introduced, debt constraints will be highly recommended for preventing over-borrowing. The same holds also for a borrowing competence of the local level with limited tax assignments. Only in case of a high fiscal autonomy of the local level, an independent borrowing competence can be seen as adequate.

6. APPENDIX

6.1.°Conditions for the Propositions 1 and 2:

Necessary and sufficient conditions for local asymptotically stability of the system (9a'-9b') are that the Jacobian matrix $\mathbf{J}$ must have a negative trace (trace $\mathbf{J} < 0$) and a positive determinant $(\det \mathbf{J} > 0)$, where $\mathbf{J}$ is given by

$$\mathbf{J} = \begin{pmatrix} \tilde{E}_k^1 & \tilde{E}_b^1 \\ \tilde{E}_k^2 & \tilde{E}_b^2 \end{pmatrix} \tag{17}$$

with the partial derivatives

$$\tilde{E}_k^1(k,b) = \left(1-(1-s)(1-\tau)-g\right) f_k - (1-s)(1-\tau)(1+\beta)\, f_{kk}\, b - n$$

$$\tilde{E}_b^1(k,b) = -(1-s)(1-\tau)(1+\beta)\, f_k < 0$$

$$\tilde{E}_k^2(k,b) = (g-\tau)(1+\beta)\, f_k + (1-\tau)(1+\beta)\, f_{kk}\, b$$

$$\tilde{E}_b^2(k,b) = (1-\tau)(1+\beta)\, f_k - n.$$

6.2.°Proof of Proposition 1:

The assumption of $(g-\tau<0)$ and a positive deficit $b>0$ requires a negative denominator of the bb-isocline, which is equivalent to $\tilde{E}_b^2 > 0$. Because of the stability requirement trace $\mathbf{J} < 0$, $\tilde{E}_k^1 < 0$ follows necessarily. However, this will contradict the stability condition $\det \mathbf{J} > 0$. Obviously one of the stability conditions will be violated.

6.3.°Proof of Proposition 2:

Proposition 1 set the requirements of $g-\tau>0$ and a positive denominator of the bb isocline for a positive public debt $b>0$ which implies $F_b^b < 0$.

With

$$\det \mathbf{J} = \tilde{E}_k^k \tilde{E}_b^b - \tilde{E}_b^k \tilde{E}_b^b - \tilde{E}_b^b \tilde{E}_k^b$$

the stability condition $\det \mathbf{J} > 0$ is equivalent to

$$\left.\frac{db}{dk}\right|_{kk} \equiv -\frac{\tilde{E}_k^k}{\tilde{E}_b^k} < -\frac{\tilde{E}_k^b}{\tilde{E}_b^b} \equiv \left.\frac{db}{dk}\right|_{bb}.$$

Simultaneously, the trace $\mathbf{J}$ is negative, iff

$$\left.\frac{db}{dk}\right|_{kk} < \frac{\tilde{E}_b^b}{\tilde{E}_b^k} > 0.$$

6.4.°Conditions for the Propositions 3 and 4:

Referring to Fuller (1968), necessary and sufficient conditions for local asymptotically stability of a steady state of the dynamic system (9a'-9c') with the Jacobian matrix

$$\mathbf{J} = \begin{pmatrix} \tilde{E}_k^1 & \tilde{E}_b^1 & \tilde{E}_\lambda^1 \\ \tilde{E}_k^2 & \tilde{E}_b^2 & \tilde{E}_\lambda^2 \\ \tilde{E}_k^3 & \tilde{E}_b^3 & \tilde{E}_\lambda^3 \end{pmatrix} := \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \tag{18}$$

are

$$\operatorname{trace}(\mathbf{J}) = a_{11} + a_{22} + a_{33} < 0\,, \tag{19a}$$

$$\det(\mathbf{J}) = a_{33}\left(a_{11}a_{22} + a_{21}a_{12}\right) < 0\,, \tag{19b}$$

$$\det(Fuller) = \left(a_{11} + a_{22}\right)\left[\left(a_{11} + a_{22} + a_{33}\right)a_{33} + \left(a_{11}a_{22} + a_{21}a_{12}\right)\right] < 0\,, \tag{19c}$$

whereby the Fuller matrix is

$$Fuller := \begin{pmatrix} a_{11} + a_{22} & a_{23} & -a_{13} \\ a_{32} & a_{33} + a_{11} & a_{12} \\ -a_{31} & a_{21} & a_{33} + a_{22} \end{pmatrix}.$$

6.4.°Proof of Proposition 3:

From (19b) follows, that a33 can only be positive if $\left(a_{11}a_{22} + a_{21}a_{12}\right) < 0$ hold. Then using (19a) must $\left(a_{11} + a_{22}\right) < 0$ also be fulfilled. However, this violates (19c). Thus, $a_{33} = \tilde{E}_\lambda^3 < 0$ must hold.

6.5.°Proof of Proposition 4:

From Proposition 3 follows, that stability in a federation requires $a_{33} < 0$. Using (19b), it is for federal stability necessary that $\left(a_{11}a_{22} + a_{21}a_{12}\right) > 0$ hold. Together with (19a) and (19c), $\left(a_{11} + a_{22}\right) < 0$ must be fulfilled. However, these are the conditions of stability in the centralized model [see Conditions for the Propositions 1 and 2].

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Notes

1 For other economic arguments to consider the flow instead of the stock see [Barro, 1990: S107; Barro and Sala-I-Martin, 1992: 650 f.].
2 Partial derivatives with respect to k or b are indicated by subscripts.
3 Government bonds and private investments are perfect substitutes. This implies the interest rate on bonds being equal to the private rental rate of capital.
4 An over dot indicates the derivative with respect to time.
5 For an extensively explanation of diagrammatic presentation in the case of perfect capital substitutes, see Wenzel (2001).